2. Mathematics with Maple: the Basics

2.1 Introduction

```
> 1 + 2;
```

$$3$$

```
> 1 + 3/2;
```

$$\frac{5}{2}$$

```
> 2*(3+1/3)/(5/3-4/5);
```

$$\frac{100}{13}$$

```
> 2.8754/2;
```

$$1.437700000$$

```
> 1 + 1/2;
```

$$\frac{3}{2}$$

2.2 Numerical Computations

Integer computations

```
> 1 + 2;
```

$$3$$

```
> 75 - 3;
```

$$72$$

```
> 5*3;
```

$$15$$

```
> 120/2;
```

$$60$$

```
> 100!;
>
```

933262154439441526816992388562667004907159682643816214685929638952175999932299156089414639761565182862536979208272237582511852109168640000000000000000000000000

```
> length(%);
```

$$158$$

```
> ifactor(60);
```

$$(2)^2\,(3)\,(5)$$

```
> igcd(123, 45);
```

$$3$$

```
> iquo(25,3);
```

$$8$$

```
> isprime(18002676583);
```

true

Exact Arithmetic - Rationals, Irrationals and Constants

```
> 1/2 + 1/3;
```

$$\frac{5}{6}$$

```
> Pi;
```

$$\pi$$

```
> evalf(Pi, 100);
```

3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117068

```
> 1/3;
```

$$\frac{1}{3}$$

```
> evalf(%);
```

$$.3333333333$$

```
> 3/2*5;
```

$$\frac{15}{2}$$

```
> 1.5*5;
```

$$7.5$$

```
> sqrt(2);
```

$$\sqrt{2}$$

```
> sqrt(3)^2;
```

$$3$$

```
> Pi;
```

$$\pi$$

```
> sin(Pi);
```

$$0$$

```
> exp(1);
```

$$\mathbf{e}$$

```
> ln(exp(5));
```

$$5$$

Floating-Point Approximations

```
> evalf(Pi);
```

$$3.141592654$$

```
> evalf(Pi, 200);
```

3.14159265358979323846264338327950288419716939937510\
58209749445923078164062862089986280348253421170679 82\
14808651328230664709384460955058223172535940812848 11\
17450284102701938521105559644622948954930382 0

```
> 1/3 + 1/4 + 1/5.3;
```

$$.7720125786$$

```
> sin(0.2);
```

$$.1986693308$$

```
> Digits := 20;
```

$$Digits := 20$$

```
> sin(0.2);
```

$$.19866933079506121546$$

Arithmetic with Special Numbers

```
> (2 + 5*I) + (1 - I);
```

$$3 + 4\,I$$

```
> (1 + I)/(3 - 2*I);
```

$$\frac{1}{13} + \frac{5}{13} I$$

```
> convert(247, binary);
```

$$11110111$$

```
> convert(1023, hex);
```

$$\mathit{3FF}$$

```
> convert(17, base, 3);
```

$$[2, 2, 1]$$

```
> 27 mod 4;
```

$$3$$

```
> mods(27,4);
```

$$-1$$

```
> modp(27,4);
```

$$3$$

Mathematical Functions

```
> sin(Pi/4);
```

$$\frac{1}{2}\sqrt{2}$$

```
> ln(1);
```

$$0$$

```
> ln(Pi);
```

$$\ln(\pi)$$

2.3 Basic Symbolic Computations

```
> (1 + x)^2;
```

$$(1+x)^2$$

```
> (1 + x) + (3 - 2*x);
```

$$4-x$$

```
> expand((1 + x)^2);
```

$$1+2\,x+x^2$$

```
> factor(%);
```

$$(1+x)^2$$

```
> Diff(sin(x), x);
```

$$\frac{\partial}{\partial x}\sin(x)$$

```
> value(%);
```

$$\cos(x)$$

```
> Sum(n^2, n);
```

$$\sum_{n} n^2$$

```
> value(%);
```

$$\frac{1}{3}n^3-\frac{1}{2}n^2+\frac{1}{6}n$$

```
> rem(x^3+x+1, x^2+x+1, x);
```

$$2+x$$

```
> series(sin(x), x=0, 10);
```

$$x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7+\frac{1}{362880}x^9+\mathrm{O}(x^{10})$$

2.4 Assigning Names to Expressions

General syntax: **`name := expression;`**

```
> var := x;
```

$$var := x$$

```
> term := x*y;
```

$$term := x\,y$$

```
> eqns := x = y + 2;
```

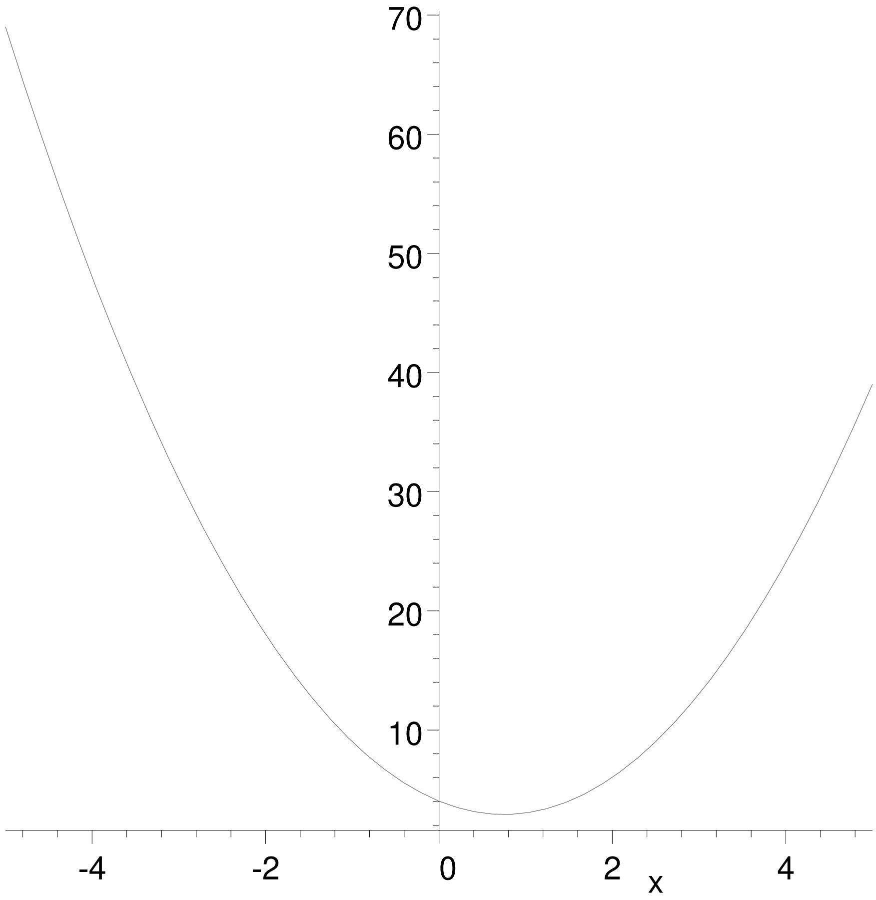
$$eqns := x = y + 2$$

Defining functions

```
> f := x -> 2*x^2 - 3*x + 4;
```

$$f := x \to 2\,x^2 - 3\,x + 4$$

```
> plot(f(x), x = -5 .. 5);
```




```
> f := x-> x^2;
```

$$f := x \rightarrow x^2$$

```
> f(5);
```

$$25$$

```
> f(y+1);
```

$$(y+1)^2$$

Protected Names

Note that Pi is NOT a protected name in Maple V.5 !

```
> # Don't try this at home!! Pi := 3.14;
> set := {1, 2, 3};
Error, attempting to assign to `set` which is
protected
```

2.5 More Basic Types of Maple Objects

Expression Sequences

```
> 1, 2, 3, 4;
```

$$1, 2, 3, 4$$

```
> x, y, z, w;
```

$$x, y, z, w$$

```
> a.b;
```

$$ab$$

```
> S := 1, 2, 3, 4;
```

$$S := 1, 2, 3, 4$$

```
> a.S;
```

$$a1, a2, a3, a4$$

Lists

```
> data_list := [1, 2, 3, 4, 5];
```

$$data\_list := [1, 2, 3, 4, 5]$$

```
> polynomials := [x^2+3, x^2+3*x-1, 2*x];
```

$$polynomials := [x^2 + 3, x^2 + 3\,x - 1, 2\,x]$$

```
> participants := [Kathy, Frank, Rene,
  Niklaus, Liz];
```

$$participants := [Kathy, Frank, Rene, Niklaus, Liz]$$

```
> [a,b,c], [b,c,a], [a,a,b,c,a];
```

$$[a, b, c], [b, c, a], [a, a, b, c, a]$$

```
> letters := [a,b,c];
```

$$letters := [a, b, c]$$

```
> letters[2];
```

$$b$$

```
> nops(letters);
```

$$3$$

```
> op(letters);
```

$$a, b, c$$

```
> letters[];
```

$$a, b, c$$

Sets

```
> data_set := {1, -1, 0, 10, 2};
```

$$data\_set := \{0, -1, 1, 2, 10\}$$

```
> unknowns := {x, y, z};
```

$$unknowns := \{x, y, z\}$$

```
> {a,b,c}, {c,b,a}, {a,a,b,c,a};
```

$$\{a, b, c\}, \{a, b, c\}, \{a, b, c\}$$

```
> {1,2,2.0};
```

$$\{1, 2, 2.0\}$$

```
> {a,b,c} union {c,d,e};
```

$$\{a, b, c, d, e\}$$

```
> {1,2,3,a,b,c} intersect {0,1,y,a};
```

$$\{1, a\}$$

```
> nops(%);
```

$$2$$

```
> op( {1,2,3,a,b} );
```

$$1, 2, 3, a, b$$

```
> numbers := {0, Pi/3, Pi/2, Pi};
```

$$numbers := \{0, \pi, \frac{1}{3}\pi, \frac{1}{2}\pi\}$$

Operations on Sets and Lists

```
> participants := [Kate, Tom, Steve];
```

$$participants := [Kate, Tom, Steve]$$

```
> member(Tom, participants);
```

$$true$$

```
> data_set := {5, 6, 3, 7};
```

$$data\_set := \{3, 5, 6, 7\}$$

```
> member(2, data_set);
```

$$false$$

```
> participants := [Kate, Tom, Steve];
```

$$participants := [Kate, Tom, Steve]$$

```
> participants[2];
```

$$Tom$$

```
> empty_set := {};
```

$$empty\_set := \{\ \}$$

```
> empty_list := [];
```

$$empty\_list := [\ ]$$

```
> old_set := {2,3,4} union {};
```

$$old\_set := \{2, 3, 4\}$$

```
> new_set := old_set union {2, 5};
```

$$new\_set := \{2, 3, 4, 5\}$$

```
> third_set := old_set minus {2, 5};
```

$$third\_set := \{3, 4\}$$

Arrays

```
> squares := array(1..3);
```

$$squares := \mathrm{array}(1 .. 3, [\ ])$$

```
> squares[1] := 1; squares[2] := 2^2;
  squares[3] := 3^2;
```

$$squares_1 := 1$$

$$squares_2 := 4$$

$$squares_3 := 9$$

```
> cubes := array(1..3, [1,8,27]);
```

$$cubes := [1, 8, 27]$$

```
> squares[2];
```

$$4$$

```
> squares;
```

$$squares$$

```
> print(squares);
```

$$[1, 4, 9]$$

```
> pwrs := array(1..3, 1..3);
```

$$pwrs := \text{array}(1 .. 3, 1 .. 3, [\ ])$$

```
> pwrs[1,1] := 1; pwrs[1,2] := 1; pwrs[1,3]
  := 1;
```

$$pwrs_{1,\,1} := 1$$

$$pwrs_{1,\,2} := 1$$

$$pwrs_{1,\,3} := 1$$

```
> pwrs[2,1] := 2: pwrs[2,2] := 4: pwrs[2,3]
  := 8:
> pwrs[3,1] := 3: pwrs[3,2] := 9: pwrs[3,3]
  := 27:
> print(pwrs);
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

```
> pwrs[2,3];
```

$$8$$

```
> array3 := array( 1..2, 1..2, 1..2,
> [[[1,2],[3,4]],[[5,6],[7,8]]]);
```

$$array3 := \mathrm{array}(1 .. 2, 1 .. 2, 1 .. 2, [$$

$$(1, 1, 1) = 1$$

$$(1, 1, 2) = 2$$

$$(1, 2, 1) = 3$$

$$(1, 2, 2) = 4$$

$$(2, 1, 1) = 5$$

$$(2, 1, 2) = 6$$

$$(2, 2, 1) = 7$$

$$(2, 2, 2) = 8$$

$$])$$

The `subs` Command

General syntax: **`subs( x=expr1, y=expr2, ... main expr );`**

```
> expr := z^2 + 3;
```

$$expr := z^2 + 3$$

```
> subs(z=x+y, expr);
```

$$(x + y)^2 + 3$$

```
> subs(2=9, pwrs);
```

$$pwrs$$

```
> subs(2=9, evalm(pwrs) );
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 9 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

```
> evalm(pwrs);
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

Tables (Associative Arrays)

```
> translate :=
  table([one=un,two=deux,three=trois]);
```

$translate :=$ table([

$one = un$

$two = deux$

$three = trois$

])

```
> translate[two];
```

$$deux$$

```
> Digits := 10;
```

$$Digits := 10$$

```
> earth_data := table(
  [mass=[5.976*10^24,kg],
>
  radius=[6.378164*10^6,m],
>
  circumference=[4.00752*10^7,m]]);
```

$earth\_data := \mathrm{table}([$

$mass = [.5976000000\ 10^{25}, kg]$

$radius = [.6378164000\ 10^{7}, m]$

$circumference = [.4007520000\ 10^{8}, m]$

$])$

```
> earth_data[mass];
```

$$[.5976000000\ 10^{25}, kg]$$

2.6 Expression Manipulation

The `simplify` Command

```
> expr := cos(x)^5 + sin(x)^4 + 2*cos(x)^2
> - 2*sin(x)^2 - cos(2*x);
```

$$expr := \cos(x)^5 + \sin(x)^4 + 2\cos(x)^2 - 2\sin(x)^2 - \cos(2\,x)$$

```
> simplify(expr);
```

$$\cos(x)^5 + \cos(x)^4$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2);
```

$$1 + \ln(2) + \ln(y)$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2,
  'trig');
```

$$1+\ln(2\,y)$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2,
  'ln');
```

$$\sin(x)^2+\ln(2)+\ln(y)+\cos(x)^2$$

The `siderel` *example gives a different result in Maple V.5*

The `factor` Command

```
> big_poly := x^5 - x^4 - 7*x^3 + x^2 + 6*x;
```

$$big\_poly := x^5-x^4-7\,x^3+x^2+6\,x$$

```
> factor(%);
```

$$x\,(x-1)\,(x-3)\,(x+2)\,(x+1)$$

```
> rat_expr := (x^3 - y^3)/(x^4 - y^4);
```

$$rat\_expr := \frac{x^3-y^3}{x^4-y^4}$$

```
> factor(rat_expr);
```

$$\frac{x^2+x\,y+y^2}{(y+x)\,(x^2+y^2)}$$

The `expand` Command

```
> expand((x+1)*(x+2));
```

$$x^2+3\,x+2$$

```
> expand(sin(x+y));
```

$$\sin(y)\cos(x)+\cos(y)\sin(x)$$

```
> expand(exp(a+ln(b)));
```

$$\mathbf{e}^{a}\, b$$

```
> expand((x+1)*(y+z), x+1);
```

$$(x+1)\,y+(x+1)\,z$$

The `convert` Command

```
> convert(cos(x), exp);
```

$$\frac{1}{2}\,\mathbf{e}^{(I\,x)}+\frac{1}{2}\,\frac{1}{\mathbf{e}^{(I\,x)}}$$

```
> convert(exp(x)/2 + exp(-x)/2, trig);
```

$$\cosh(x)$$

```
> A := array(1..2, 1..2, [[a,b], [c,d]]);
```

$$A:=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

```
> convert(A,'listlist');
```

$$[[a,b],[c,d]]$$

```
> convert(A,'set');
```

$$\{a,b,c,d\}$$

```
> convert(%,list);
```

$$[a,b,c,d]$$

The `normal` Command

```
> rat_expr_2 := (x^2 - y^2)/(x - y)^3;
```

$$rat\_expr\_2 := \frac{x^2 - y^2}{(-y + x)^3}$$

```
> normal(rat_expr_2);
```

$$\frac{y + x}{(-y + x)^2}$$

```
> normal(rat_expr_2, 'expanded');
```

$$\frac{y + x}{y^2 - 2\,x\,y + x^2}$$

The `combine` Command

```
> combine(exp(x)^2*exp(y),exp);
```

$$\mathbf{e}^{(2\,x + y)}$$

```
> combine((x^a)^2,power);
```

$$x^{(2\,a)}$$

The `map` Command

```
> f := 'f';
```

$$f := f$$

```
> map( f, [a,b,c] );
```

$$[\mathrm{f}(a), \mathrm{f}(b), \mathrm{f}(c)]$$

```
> data_list := [0, Pi/2, 3*Pi/2, 2*Pi];
```

$$\mathit{data\_list} := \left[0, \frac{1}{2}\pi, \frac{3}{2}\pi, 2\pi\right]$$

```
> map(sin,data_list);
```

$$[0, 1, -1, 0]$$

```
> map(f, [a,b,c], x, y);
```

$$[\mathrm{f}(a, x, y), \mathrm{f}(b, x, y), \mathrm{f}(c, x, y)]$$

```
> fcn_list := [sin(x), ln(x), x^2];
```

$$\mathit{fcn\_list} := [\sin(x), \ln(x), x^2]$$

```
> map(Diff,fcn_list,x);
```

$$\left[\frac{\partial}{\partial x}\sin(x), \frac{\partial}{\partial x}\ln(x), \frac{\partial}{\partial x}x^2\right]$$

```
> map(value,%);
```

$$\left[\cos(x), \frac{1}{x}, 2x\right]$$

```
> map(x->x^2, [-1,0,1,2,3]);
```

$$[1, 0, 1, 4, 9]$$

The `lhs` and `rhs` Commands

```
> eqn1 := x+y=z+3;
```

$$eqn1 := y + x = z + 3$$

```
> lhs(eqn1);
```

$$y + x$$

```
> rhs(eqn1);
```

$$z + 3$$

The `numer` and `denom` Commands

```
> numer(3/4);
```

$$3$$

```
> denom(1/(1 + x));
```

$$x + 1$$

The `nops` and `op` Commands

```
> nops(x^2);
```

$$2$$

```
> nops(x+y);
```

$$2$$

```
> op(x^2);
```

$$x, 2$$

```
> op(1,x^2);
```

$$x$$

```
> op(1..2,x+y+z+w);
```

$$x, y$$

Common Questions about Expression Manipulation

```
> expr := a^3*b^2;
```

$$expr := a^3\, b^2$$

```
> subs(a*b=5,expr);
```

$$a^3\, b^2$$

```
> simplify(expr, {a*b=5});
```

$$25\, a$$

```
> expr2 := cos(x)*(sec(x) - cos(x));
```

$$expr2 := \cos(x)\,(\sec(x) - \cos(x))$$

```
> simplify(%);
```

$$1 - \cos(x)^2$$

```
> simplify(%,{1-cos(x)^2=sin(x)^2});
```

$$\sin(x)^2$$

```
> x^19 - x;
```

$$x^{19} - x$$

```
> factor(%);
```

$$x\,(x-1)\,(x^2+x+1)\,(x^6+x^3+1)\,(x+1)\,(1-x+x^2)\,(1-x^3+x^6)$$

```
> 2*(x + y);
```

$$2\,y+2\,x$$

```
> expr3 := 2*(x + y);
```

$$expr3 := 2\,y+2\,x$$

```
> subs( 2=two, expr3 );
```

$$y\;two+x\;two$$

```
> factor(%);
```

$$two\,(y+x)$$