2. Mathematics with Maple: the Basics

2.1 Introduction

```
> 1 + 2;
```

$$3$$

```
> 1 + 3/2;
```

$$\frac{5}{2}$$

```
> 2*(3+1/3)/(5/3-4/5);
```

$$\frac{100}{13}$$

```
> 2.8754/2;
```

$$1.437700000$$

```
> 1 + 1/2;
```

$$\frac{3}{2}$$

2.2 Numerical Computations

Integer computations

```
> 1 + 2;
```

$$3$$

```
> 75 - 3;
```

$$72$$

```
> 5*3;
```

$$15$$

```
> 120/2;
```

$$60$$

```
> 100!;
```

$$933262154439441526816992388562667004907159682643816214685929638952175999932299156089414639761565182862536979208272237582511852109168640000000000000000000000$$

```
> length(%);
```

$$158$$

```
> ifactor(60);
```

$$(2)^2\ (3)\ (5)$$

```
> igcd(123, 45);
```

$$3$$

```
> iquo(25,3);
```

$$8$$

```
> isprime(18002676583);
```

$$true$$

Exact Arithmetic - Rationals, Irrationals and Constants

```
> 1/2 + 1/3;
```

$$\frac{5}{6}$$

```
> Pi;
```

$$\pi$$

```
> evalf(Pi, 100);
```

3.14159265358979323846264338327950288419716939937\
510582097494459230781640628620899862803482534211 7\
068

```
> 1/3;
```

$$\frac{1}{3}$$

```
> evalf(%);
```

$$0.3333333333$$

```
> 3/2*5;
```

$$\frac{15}{2}$$

```
> 1.5*5;
```

$$7.5$$

```
> sqrt(2);
```

$$\sqrt{2}$$

```
> sqrt(3)^2;
```

$$3$$

```
> Pi;
```

$$\pi$$

```
> sin(Pi);
```

$$0$$

```
> exp(1);
```

$$\mathbf{e}$$

```
> ln(exp(5));
```

$$5$$

Floating-Point Approximations

```
> evalf(Pi);
```

$$3.141592654$$

```
> evalf(Pi, 200);
```

$$3.14159265358979323846264338327950288419716939937\
510582097494459230781640628620899862803482534211 7\
067982148086513282306647093844609550582231725359 4\
081284811174502841027019385211055596446229489549 3\
03820$$

```
> 1/3 + 1/4 + 1/5.3;
```

$$0.7720125786$$

```
> sin(0.2);
```

$$0.1986693308$$

```
Digits := 20;
```

$$Digits := 20$$

```
> sin(0.2);
```

$$0.19866933079506121546$$

Arithmetic with Special Numbers

```
> (2 + 5*I) + (1 - I);
```

$$3 + 4\,I$$

```
> (1 + I)/(3 - 2*I);
```

$$\frac{1}{13} + \frac{5}{13} I$$

```
> convert(247, binary);
```

$$11110111$$

```
> convert(1023, hex);
```

$$3FF$$

```
> convert(17, base, 3);
```

$$[2, 2, 1]$$

```
> 27 mod 4;
```

$$3$$

```
> mods(27,4);
```

$$-1$$

```
> modp(27,4);
```

$$3$$

Mathematical Functions

```
> sin(Pi/4);
```

$$\frac{\sqrt{2}}{2}$$

```
> ln(1);
```

$$0$$

```
> ln(Pi);
```

$$\ln(\pi)$$

2.3 Basic Symbolic Computations

```
> (1 + x)^2;
```

$$(1+x)^2$$

```
> (1 + x) + (3 - 2*x);
```

$$4-x$$

```
> expand((1 + x)^2);
```

$$1 + 2\,x + x^2$$

```
> factor(%);
```

$$(1 + x)^2$$

```
> Diff(sin(x), x);
```

$$\frac{d}{dx}\sin(x)$$

```
> value(%);
```

$$\cos(x)$$

```
> Sum(n^2, n);
```

$$\sum_{n} n^2$$

```
> value(%);
```

$$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$$

```
> rem(x^3+x+1, x^2+x+1, x);
```

$$2 + x$$

```
> series(sin(x), x=0, 10);
```

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 + \mathrm{O}(x^{10})$$

2.4 Assigning Names to Expressions

General syntax: `name := expression;`

```
> var := x;
```

$$var := x$$

```
> term := x*y;
```

$$term := x\,y$$

```
> eqns := x = y + 2;
```

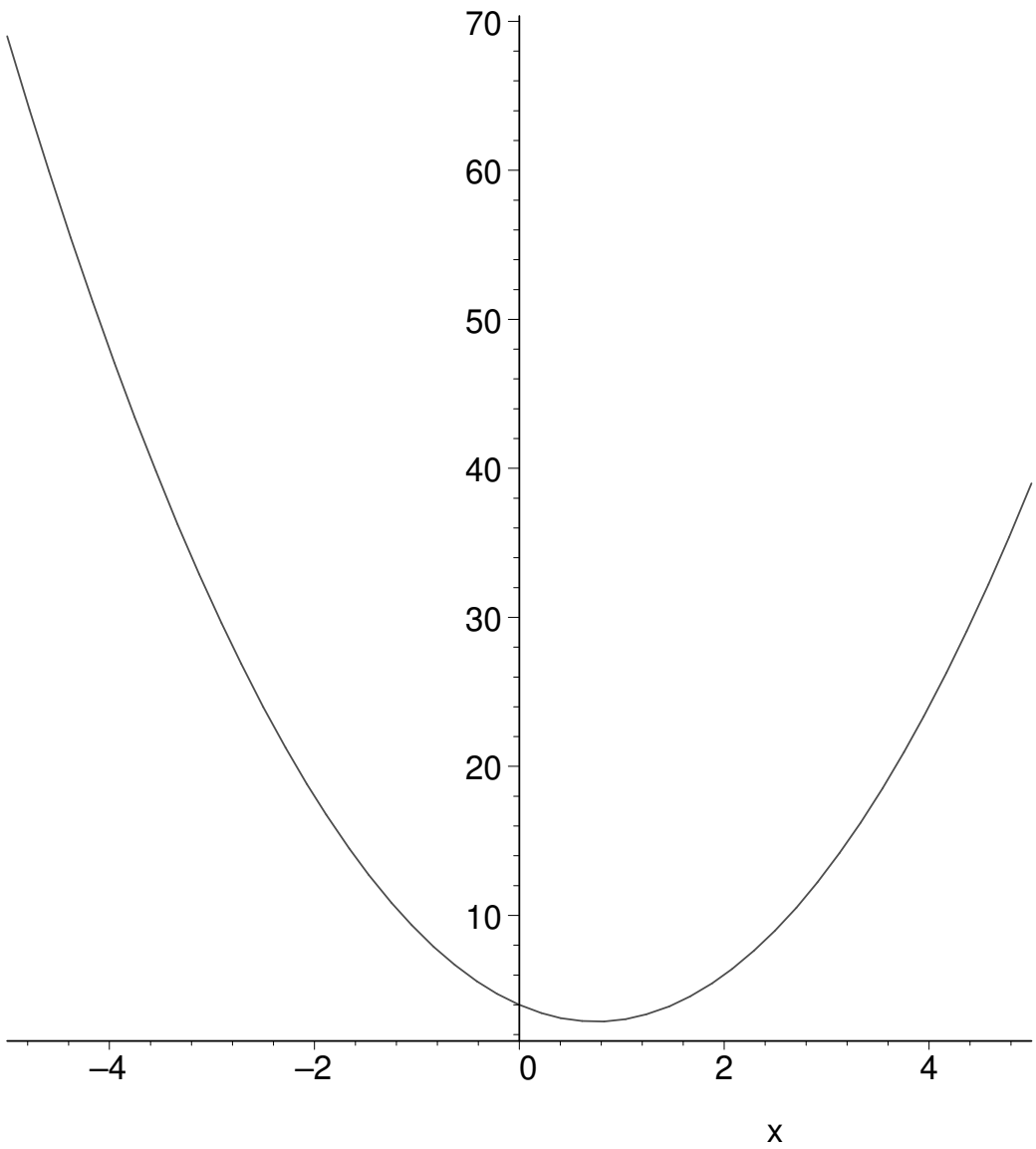
$$eqns := x = y + 2$$

Defining functions

```
> f := x -> 2*x^2 - 3*x + 4;
```

$$f := x \to 2\,x^2 - 3\,x + 4$$

```
> plot(f(x), x = -5 .. 5);
```



```
> f := x-> x^2;
```

$$f := x \to x^2$$

```
> f(5);
```

$$25$$

```
> f(y+1);
```

$$(y+1)^2$$

Protected Names

```
> Pi := 3.14;

Error, attempting to assign to `Pi` which is protected
```

```
> set := {1, 2, 3};

Error, attempting to assign to `set` which is
protected
```

2.5 More Basic Types of Maple Objects

Expression Sequences

```
> 1, 2, 3, 4;
```

$$1, 2, 3, 4$$

```
> x, y, z, w;
```

$$x, y, z, w$$

```
> a||b;
```

$$ab$$

```
> S := 1, 2, 3, 4;
```

$$S := 1, 2, 3, 4$$

```
> a||S;
```

$$a1, a2, a3, a4$$

Lists

```
> data_list := [1, 2, 3, 4, 5];
```

$$data_list := [1, 2, 3, 4, 5]$$

```
> polynomials := [x^2+3, x^2+3*x-1, 2*x];
```

$$polynomials := [x^2 + 3, x^2 + 3\,x - 1, 2\,x]$$

```
> participants := [Kathy, Frank, Rene,
  Niklaus, Liz];
```

$$participants := [Kathy, Frank, Rene, Niklaus, Liz]$$

```
> [a,b,c], [b,c,a], [a,a,b,c,a];
```

$$[a, b, c], [b, c, a], [a, a, b, c, a]$$

```
> letters := [a,b,c];
```

$$letters := [a, b, c]$$

```
> letters[2];
```

$$b$$

```
> nops(letters);
```

$$3$$

```
> op(letters);
```

$$a, b, c$$

```
> letters[];
```

$$a, b, c$$

Sets

```
> data_set := {1, -1, 0, 10, 2};
```

$$data_set := \{-1, 0, 1, 2, 10\}$$

```
> unknowns := {x, y, z};
```

$$unknowns := \{x, y, z\}$$

```
> {a,b,c}, {c,b,a}, {a,a,b,c,a};
```

$$\{a, b, c\}, \{a, b, c\}, \{a, b, c\}$$

```
> {1,2,2.0};
```

$$\{1, 2, 2.0\}$$

```
> {a,b,c} union {c,d,e};
```

$$\{a, b, c, d, e\}$$

```
> {1,2,3,a,b,c} intersect {0,1,y,a};
```

$$\{1, a\}$$

```
> nops(%);
```

$$2$$

```
> op( {1,2,3,a,b} );
```

$$1, 2, 3, a, b$$

```
> numbers := {0, Pi/3, Pi/2, Pi};
```

$$numbers := \{0, \pi, \frac{\pi}{3}, \frac{\pi}{2}\}$$

```
> map(g, numbers);
```

$$\{g(0), g(\pi), g\left(\frac{\pi}{3}\right), g\left(\frac{\pi}{2}\right)\}$$

```
> map(sin, numbers);
```

$$\{0, 1, \frac{\sqrt{3}}{2}\}$$

Operations on Sets and Lists

```
> participants := [Kate, Tom, Steve];
```

$$participants := [Kate, Tom, Steve]$$

```
> member(Tom, participants);
```

$$true$$

```
> data_set := {5, 6, 3, 7};
```

$$data_set := \{3, 5, 6, 7\}$$

```
> member(2, data_set);
```

$$false$$

```
> participants := [Kate, Tom, Steve];
```

$$participants := [Kate, Tom, Steve]$$

```
> participants[2];
```

$$Tom$$

```
> empty_set := {};
```

$$empty_set := \{\ \}$$

```
> empty_list := [];
```

$$empty_list := [\ ]$$

```
> old_set := {2,3,4} union {};
```

$$old_set := \{2, 3, 4\}$$

```
> new_set := old_set union {2, 5};
```

$$new_set := \{2, 3, 4, 5\}$$

```
> third_set := old_set minus {2, 5};
```

$$third_set := \{3, 4\}$$

Arrays

```
> squares := array(1..3);
```

$$squares := \mathrm{array}(1 .. 3, [\ ])$$

```
> squares[1] := 1; squares[2] := 2^2;
  squares[3] := 3^2;
```

$$squares_1 := 1$$

$$squares_2 := 4$$

$$squares_3 := 9$$

```
> cubes := array(1..3, [1,8,27]);
```

$$cubes := [1, 8, 27]$$

```
> squares[2];
```

$$4$$

```
> squares;
```

$$squares$$

```
> print(squares);
```

$$[1, 4, 9]$$

```
> pwrs := array(1..3, 1..3);
```

$$pwrs := \mathrm{array}(1 .. 3, 1 .. 3, [\ ])$$

```
> pwrs[1,1] := 1; pwrs[1,2] := 1; pwrs[1,3]
  := 1;
```

$$pwrs_{1,1} := 1$$

$$pwrs_{1,2} := 1$$

$$pwrs_{1,3} := 1$$

```
> pwrs[2,1] := 2: pwrs[2,2] := 4: pwrs[2,3]
  := 8:
> pwrs[3,1] := 3: pwrs[3,2] := 9: pwrs[3,3]
  := 27:
> print(pwrs);
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

```
> pwrs[2,3];
```

$$8$$

```
> array3 := array( 1..2, 1..2, 1..2,
> [[[1,2], [3,4]], [[5,6], [7,8]]] );
```

$$array3 := \text{array}(1 .. 2, 1 .. 2, 1 .. 2, [$$

$$(1,1,1) = 1$$

$$(1,1,2) = 2$$

$$(1,2,1) = 3$$

$$(1,2,2) = 4$$

$$(2,1,1) = 5$$

$$(2,1,2) = 6$$

$$(2,2,1) = 7$$

$$(2,2,2) = 8$$

```
    ])
```

The `subs` Command

General syntax: `subs( x=expr1, y=expr2, ... main expr );`

```
> expr := z^2 + 3;
```

$$expr := z^2 + 3$$

```
> subs(z=x+y, expr);
```

$$(x+y)^2 + 3$$

```
> subs(2=9, pwrs);
```

$$pwrs$$

```
> subs(2=9, evalm(pwrs) );
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 9 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

```
> evalm(pwrs);
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

Tables (Associative Arrays)

```
> translate :=
  table([one=un,two=deux,three=trois]);
```

$$translate := \mathrm{table}([three = trois, one = un, two = deux])$$

```
> translate[two];
```

$$deux$$

```
> Digits := 10;
```

$$Digits := 10$$

```
> earth_data := table(
  [mass=[5.976*10^24,kg],

  radius=[6.378164*10^6,m],

  circumference=[4.00752*10^7,m]]);
```

$$earth_data := \mathrm{table}([mass = [0.5976000000\ 10^{25}, kg],$$
$$radius = [0.6378164000\ 10^{7}, m],$$
$$circumference = [0.4007520000\ 10^{8}, m]$$
$$])$$

```
> earth_data[mass];
```

$$[0.5976000000\ 10^{25}, kg]$$

2.6 Expression Manipulation

The `simplify` Command

```
> expr := cos(x)^5 + sin(x)^4 + 2*cos(x)^2
  - 2*sin(x)^2 - cos(2*x);
```

$$expr := \cos(x)^5 + \sin(x)^4 + 2\cos(x)^2 - 2\sin(x)^2 - \cos(2x)$$

```
> simplify(expr);
```

$$\cos(x)^4(\cos(x)+1)$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2);
```

$$1 + \ln(2) + \ln(y)$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2,
  'trig');
```

$$1 + \ln(2y)$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2,
  'ln');
```

$$\sin(x)^2 + \ln(2) + \ln(y) + \cos(x)^2$$

The `siderel` example gives a different result in Maple 8

The `factor` Command

```
> big_poly := x^5 - x^4 - 7*x^3 + x^2 + 6*x;
```

$$big_poly := x^5 - x^4 - 7\,x^3 + x^2 + 6\,x$$

```
> factor(big_poly);
```

$$x\,(x-1)\,(x-3)\,(2+x)\,(1+x)$$

```
> rat_expr := (x^3 - y^3)/(x^4 - y^4);
```

$$rat_expr := \frac{x^3 - y^3}{x^4 - y^4}$$

```
> factor(rat_expr);
```

$$\frac{x^2 + x\,y + y^2}{(x+y)\,(x^2+y^2)}$$

The `expand` Command

```
> expand((x+1)*(x+2));
```

$$x^2 + 3\,x + 2$$

```
> expand(sin(x+y));
```

$$\sin(x)\cos(y) + \cos(x)\sin(y)$$

```
> expand(exp(a+ln(b)));
```

$$\mathbf{e}^a\,b$$

```
> expand((x+1)*(y+z), x+1);
```

$$(1+x)\,y+(1+x)\,z$$

The `convert` Command

```
> convert(cos(x), exp);
```

$$\frac{1}{2}\,\mathbf{e}^{(x\,I)}+\frac{1}{2}\,\frac{1}{\mathbf{e}^{(x\,I)}}$$

```
> convert(exp(x)/2 + exp(-x)/2, trig);
```

$$\cosh(x)$$

```
> A := array(1..2, 1..2, [[a,b], [c,d]]);
```

$$A := \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

```
> convert(A,'listlist');
```

$$[[a, b], [c, d]]$$

```
> convert(A,'set');
```

$$\{a, b, c, d\}$$

```
> convert(%,list);
```

$$[a, b, c, d]$$

The `normal` Command

```
> rat_expr_2 := (x^2 - y^2)/(x - y)^3;
```

$$rat_expr_2 := \frac{x^2 - y^2}{(-y + x)^3}$$

```
> normal(rat_expr_2);
```

$$\frac{x + y}{(-y + x)^2}$$

```
> normal(rat_expr_2, 'expanded');
```

$$\frac{x + y}{y^2 - 2\,x\,y + x^2}$$

The `combine` Command

```
> combine(exp(x)^2*exp(y),exp);
```

$$\mathbf{e}^{(2\,x + y)}$$

```
> combine((x^a)^2,power);
```

$$x^{(2\,a)}$$

The `expr := ... combine(expr)` ... example does not work as advertised in Maple 8

The `map` Command

Be careful to "reset" the symbol `f`

```
> f := 'f';
```

$$f := f$$

```
> map( f, [a,b,c] );
```

$$[\mathrm{f}(a), \mathrm{f}(b), \mathrm{f}(c)]$$

```
> data_list := [0, Pi/2, 3*Pi/2, 2*Pi];
```

$$data_list := \left[0, \frac{\pi}{2}, \frac{3\,\pi}{2}, 2\,\pi\right]$$

```
> map(sin,data_list);
```

$$[0, 1, -1, 0]$$

```
> map(f, [a,b,c], x, y);
```

$$[\mathrm{f}(a, x, y), \mathrm{f}(b, x, y), \mathrm{f}(c, x, y)]$$

```
> fcn_list := [sin(x), ln(x), x^2];
```

$$fcn_list := [\sin(x), \ln(x), x^2]$$

```
> map(Diff,fcn_list,x);
```

$$\left[\frac{d}{dx}\sin(x), \frac{d}{dx}\ln(x), \frac{d}{dx}(x^2)\right]$$

```
> map(value,%);
```

$$\left[\cos(x), \frac{1}{x}, 2\,x\right]$$

```
> map(x->x^2, [-1,0,1,2,3]);
```

$$[1, 0, 1, 4, 9]$$

The `lhs` and `rhs` Commands

```
> eqn1 := x+y=z+3;
```

$$eqn1 := x + y = z + 3$$

```
> lhs(eqn1);
```

$$x + y$$

```
> rhs(eqn1);
```

$$z + 3$$

The `numer` and `denom` Commands

```
> numer(3/4);
```

$$3$$

```
> denom(1/(1 + x));
```

$$1 + x$$

The `nops` and `op` Commands

```
> nops(x^2);
```

$$2$$

```
> nops(x+y);
```

$$2$$

```
> op(x^2);
```

$$x, 2$$

```
> op(1,x^2);
```

$$x$$

```
> op(2,x^2);
```

$$2$$

```
> op(1..2,x+y+z+w);
```

$$x, y$$

Common Questions about Expression Manipulation

```
> expr := a^3*b^2;
```

$$expr := a^3 b^2$$

```
> subs(a*b=5,expr);
```

$$a^3 b^2$$

```
> simplify(expr, {a*b=5});
```

$$25\, a$$

```
> expr2 := cos(x)*(sec(x) - cos(x));
```

$$expr2 := \cos(x)\,(\sec(x) - \cos(x))$$

```
> simplify(%);
```

$$1 - \cos(x)^2$$

```
> simplify(%,{1-cos(x)^2=sin(x)^2});
```

$$\sin(x)^2$$

```
> x^19 - x;
```

$$x^{19} - x$$

```
> factor(%);
```

$$x\,(x-1)\,(x^2+x+1)\,(x^6+x^3+1)\,(1+x)\,(1-x+x^2)\,(1-x^3+x^6)$$

```
> 2*(x + y);
```

$$2x+2y$$

```
> expr3 := 2*(x + y);
```

$$expr3 := 2x+2y$$

```
> subs( 2=two, expr3 );
```

$$x\,two + y\,two$$

```
> factor(%);
```

$$two\,(x+y)$$