Solutions for Quiz 9 – Week of November 12, 2007

21.13. Model: A string fixed at both ends forms standing waves.
Solve: (a) Three antinodes means the string is vibrating as the $m = 3$ standing wave. The frequency is $f_3 = 3f_1$, so the fundamental frequency is $f_1 = \frac{1}{3}$ (420 Hz) = 140 Hz. The fifth harmonic will have the frequency $f_5 = 5f_1 = 700$ Hz.

(b) The wavelength of the fundamental mode is $\lambda_1 = 2L = 1.20$ m. The wave speed on the string is $v = \lambda_1 f_1$ = (1.20 m)(140 Hz) = 168 m/s. Alternatively, the wavelength of the $n = 3$ mode is $\lambda_3 = \frac{1}{3}(2L) = 0.40$ m, from which $v = \lambda_3 f_3$ = (0.40 m)(420 Hz) = 168 m/s. The wave speed on the string is given by

$$v = \sqrt{\frac{T_S}{\mu}} \Rightarrow T_S = \mu v^2 = (0.0020 \text{ kg/m})(168 \text{ m/s})^2 = 56.4 \text{ N}$$

Assess: You must remember to use the linear density in SI units of kg/m. Also, the speed is the same for all modes, but you must use a matching λ and f to calculate the speed.

21.45. Model: The steel wire is under tension and it vibrates with three antinodes.
Visualize: Please refer to Figure P21.45.
Solve: When the spring is stretched 8.0 cm, the standing wave on the wire has three antinodes. This means $\lambda_3 = \frac{2}{3}L$ and the tension T_S in the wire is $T_S = k(0.080 \text{ m})$, where k is the spring constant. For this tension,

$$v_{\text{wire}} = \sqrt{\frac{T_S}{\mu}} \Rightarrow f\lambda_3 = \sqrt{\frac{T_S}{\mu}} \Rightarrow f = \frac{3}{2L}\sqrt{\frac{k(0.08 \text{ m})}{\mu}}$$

We will let the stretching of the spring be Δx when the standing wave on the wire displays two antinodes. This means $\lambda_2 = L$ and $T_S' = kx$. For the tension T_S',

$$v_{\text{wire}}' = \sqrt{\frac{T_S'}{\mu}} \Rightarrow f\lambda_2 = \sqrt{\frac{T_S'}{\mu}} \Rightarrow f = \frac{1}{L}\sqrt{\frac{k\Delta x}{\mu}}$$

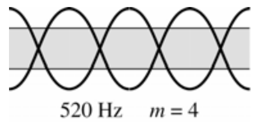
The frequency f is the same in the above two situations because the wire is driven by the same oscillating magnetic field. Now, equating the two frequency equations,

$$\frac{1}{L}\sqrt{\frac{k\Delta x}{\mu}} = \frac{3}{2L}\sqrt{\frac{k(0.080 \text{ m})}{\mu}} \Rightarrow \Delta x = 0.18 \text{ m} = 18 \text{ cm}$$

21.48. Model: A tube forms standing waves.
Solve: (a) The fundamental frequency cannot be 390 Hz because 520 Hz and 650 Hz are not integer multiples of it. But we note that the *difference* between 390 Hz and 520 Hz is 130 Hz as is the *difference* between 520 Hz and 650 Hz. We see that 390 Hz = 3 × 130 Hz = $3f_1$, 520 Hz = $4f_1$, and 650 Hz = $5f_1$. So we are seeing the third, fourth, and fifth harmonics of a tube whose fundamental frequency is 130 Hz. According to Equation 21.17, this is an open-open tube because $f_m = mf_1$ with $m = 1, 2, 3, 4, \ldots$ For an open-closed tube m has only odd values.
(b) Knowing f_1, we can now find the length of the tube:

$$L = \frac{v}{2f_1} = \frac{343 \text{ m/s}}{2(130 \text{ Hz})} = 1.32 \text{ m}$$

(c) 520 Hz is the fourth harmonic. This is a sound wave, not a wave on a string, so the wave will have four nodes and will have antinodes at the ends, as shown.



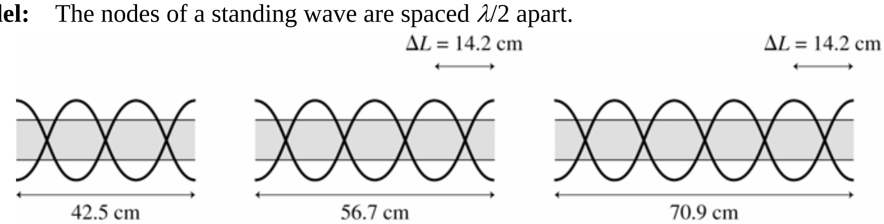
(d) With carbon dioxide, the new fundamental frequency is

$$f_1 = \frac{v}{2L} = \frac{280\ \text{m/s}}{2(1.32\ \text{m})} = 106\ \text{Hz}$$

Thus the frequencies of the $n = 3$, 4, and 5 modes are $f_3 = 3f_1 = 318$ Hz, $f_4 = 4f_1 = 424$ Hz, and $f_5 = 5f_1 = 530$ Hz.

21.50. Model: The nodes of a standing wave are spaced $\lambda/2$ apart.

Visualize:



Solve: The wavelength of the mth mode of an open-open tube is $\lambda_m = 2L/m$. Or, equivalently, the length of the tube that generates the mth mode is $L = m(\lambda/2)$. Here λ is the same for all modes because the frequency of the tuning fork is unchanged. Increasing the length of the tube to go from mode m to mode $m + 1$ requires a length change

$$\Delta L = (m+1)(\lambda/2) - m\lambda/2 = \lambda/2$$

That is, lengthening the tube by $\lambda/2$ adds an additional antinode and creates the next standing wave. This is consistent with the idea that the nodes of a standing wave are spaced $\lambda/2$ apart. This tube is first increased $\Delta L =$ 56.7 cm – 42.5 cm = 14.2 cm, then by ΔL = 70.9 cm – 56.7 cm = 14.2 cm. Thus $\lambda/2$ = 14.2 cm and thus λ = 28.4 cm = 0.284 m. Therefore the frequency of the tuning fork is

$$f = \frac{v}{\lambda} = \frac{343\ \text{m/s}}{0.284\ \text{m}} = 1208\ \text{Hz}$$