Solutions for Quiz 8 – Week of October 29, 2007

20.71. Model: We have a traveling wave radiated by the tornado siren.
Solve: (a) The power of the source is calculated as follows:

$$I_{50\text{ m}} = 0.10\text{ W/m}^2 = \frac{P_{\text{source}}}{4\pi r^2} = \frac{P_{\text{source}}}{4\pi(50\text{ m})^2} \Rightarrow P_{\text{source}} = (0.10\text{ W/m}^2)4\pi(50\text{ m})^2 = (1000\pi)\text{ W}$$

The intensity at 1000 m is

$$I_{1000\text{ m}} = \frac{P_{\text{source}}}{4\pi(1000\text{ m})^2} = \frac{(1000\pi)\text{ W}}{4\pi(1000\text{ m})^2} = 250\ \mu\text{W/m}^2$$

(b) The maximum distance is calculated as follows:

$$I = \frac{P_{\text{source}}}{4\pi r^2} \Rightarrow 1.0\times10^{-6}\text{ W/m}^2 = \frac{(1000\pi)\text{ W}}{4\pi r^2} \Rightarrow r = 15.8\text{ km}$$

20.73. Model: The bat's chirping frequency is altered by the Doppler effect. The frequency is increased as the bat approaches and it decreases as the bat recedes away.
Solve: The bat must fly away from you, so that the chirp frequency observed by you is less than 25 kHz. From Equation 20.38,

$$f_- = \frac{f_0}{1+v_S/v} \Rightarrow 20{,}000\text{ Hz} = \frac{25000\text{ Hz}}{1+\left(\dfrac{v_S}{343\text{ m/s}}\right)} \Rightarrow v_S = 85.8\text{ m/s}$$

Assess: This is a rather large speed: 85.8 m/s ≈ 180 mph. This is not possible for a bat.

20.74. Model: The sound generator's frequency is altered by the Doppler effect. The frequency increases as the generator approaches the student, and it decreases as the generator recedes from the student.
Solve: The generator's speed is

$$v_S = r\omega = r(2\pi f) = (1.0\text{ m})2\pi\left(\frac{100}{60}\text{ rev/s}\right) = 10.47\text{ m/s}$$

The frequency of the approaching generator is

$$f_+ = \frac{f_0}{1-v_S/v} = \frac{600\text{ Hz}}{1-\dfrac{10.47\text{ m/s}}{343\text{ m/s}}} = 619\text{ Hz}$$

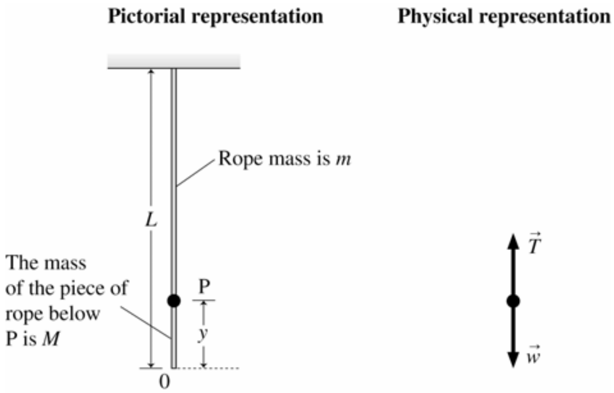
Doppler effect for the receding generator, on the other hand, is

$$f_- = \frac{f_0}{1+v_S/v} = \frac{600\text{ Hz}}{1+\dfrac{10.47\text{ m/s}}{343\text{ m/s}}} = 582\text{ Hz}$$

Thus, the highest and the lowest frequencies heard by the student are 619 Hz and 582 Hz.

20.82. Model: The wave pulse is a traveling wave on a stretched wire.

Visualize:



Solve: **(a)** At a distance y above the lower end of the rope, the point P is in static equilibrium. The upward tension in the rope must balance the weight of the rope that hangs below this point. Thus, at this point

$$T = w = Mg = (\mu y)g$$

where $\mu = m/L$ is the linear density of the entire rope. Using Equation 20.2, we get

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu y g}{\mu}} = \sqrt{gy}$$

(b) The time to travel a distance dy at y, where the wave speed is $v = \sqrt{gy}$, is

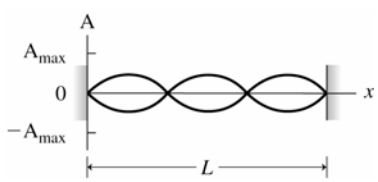
$$dt = \frac{dy}{v} = \frac{dy}{\sqrt{gy}}$$

Finding the time for a pulse to travel the length of the rope requires integrating from one end of the rope to the other:

$$\Delta t = \int_0^T dt = \int_0^L \frac{dy}{\sqrt{gy}} = \frac{1}{\sqrt{g}}\left(2\sqrt{y}\Big|_0^L\right) = \frac{2}{\sqrt{g}}\sqrt{L} \Rightarrow \Delta t = 2\sqrt{\frac{L}{g}}$$

21.36. **Model:** The wavelength of the standing wave on a string is $\lambda_m = 2L/m$, where $m = 1, 2, 3, \ldots$ We assume that 30 cm is the first place from the left end of the string where $A = A_{max}/2$.

Visualize:



Solve: The amplitude of oscillation on the string is $A(x) = A_{max}\sin kx$. Since the string is vibrating in the third harmonic, the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(2L/3)} = 3\frac{\pi}{L}$$

Substituting into the equation for the amplitude,

$$\frac{1}{2}A_{max} = A_{max}\sin\left(\frac{3\pi}{L}(0.30\text{ m})\right) \Rightarrow \sin\left(\frac{3\pi}{L}(0.30\text{ m})\right) = \frac{1}{2} \Rightarrow \frac{3\pi}{L}(0.30\text{ m}) = \frac{\pi}{6}\text{ rad} \Rightarrow L = 5.40\text{ m}$$

21.37. Model: The wavelength of the standing wave on a string vibrating at its fundamental frequency is equal to $2L$.

Solve: The amplitude of oscillation on the string is $A(x) = 2a\sin kx$, where a is the amplitude of the traveling wave and the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2L} = \frac{\pi}{L}$$

Substituting into the above equation,

$$A\left(x = \tfrac{1}{4}L\right) = 2.0 \text{ cm} = 2a\sin\left[\left(\frac{\pi}{L}\right)\left(\frac{L}{4}\right)\right] \Rightarrow 1.0 \text{ cm} = a\left(\frac{1}{\sqrt{2}}\right) \Rightarrow a = \sqrt{2} \text{ cm} = 1.41 \text{ cm}$$

21.38. Visualize: Please refer to Figure 21.4.

Solve: You can see in Figure 21.4 that the time between two successive instants when the antinodes are at maximum height is half the period, or $\tfrac{1}{2}T$. Thus $T = 2(0.25 \text{ s}) = 0.50$ s, and so

$$f = \frac{1}{T} = \frac{1}{0.50 \text{ s}} = 2.0 \text{ Hz} \Rightarrow \lambda = \frac{v}{f} = \frac{3.0 \text{ m/s}}{2.0 \text{ Hz}} = 1.50 \text{ m}$$

21.40. Model: The wave on a stretched string with both ends fixed is a standing wave.

Solve: We must distinguish between the sound wave in the air and the wave on the string. The listener hears a sound wave of wavelength $\lambda_{\text{sound}} = 40 \text{ cm} = 0.40$ m. Thus, the frequency is

$$f = \frac{v_{\text{sound}}}{\lambda_{\text{sound}}} = \frac{343 \text{ m/s}}{0.40 \text{ m}} = 857.5 \text{ Hz}$$

The violin string oscillates at the same frequency, because each oscillation of the string causes one oscillation of the air. But the *wavelength* of the standing wave on the string is very different because the wave speed on the string is not the same as the wave speed in air. Bowing a string produces sound at the string's fundamental frequency, so the wavelength of the string is

$$\lambda_{\text{string}} = \lambda_1 = 2L = 0.60 \text{ m} \Rightarrow v_{\text{string}} = \lambda_{\text{string}} f = (0.60 \text{ m})(857.5 \text{ Hz}) = 514.5 \text{ m/s}$$

The tension is the string is found as follows:

$$v_{\text{string}} = \sqrt{\frac{T_S}{\mu}} \Rightarrow T_S = \mu\left(v_{\text{string}}\right)^2 = (0.001 \text{ kg/m})(514.5 \text{ m/s})^2 = 265 \text{ N}$$