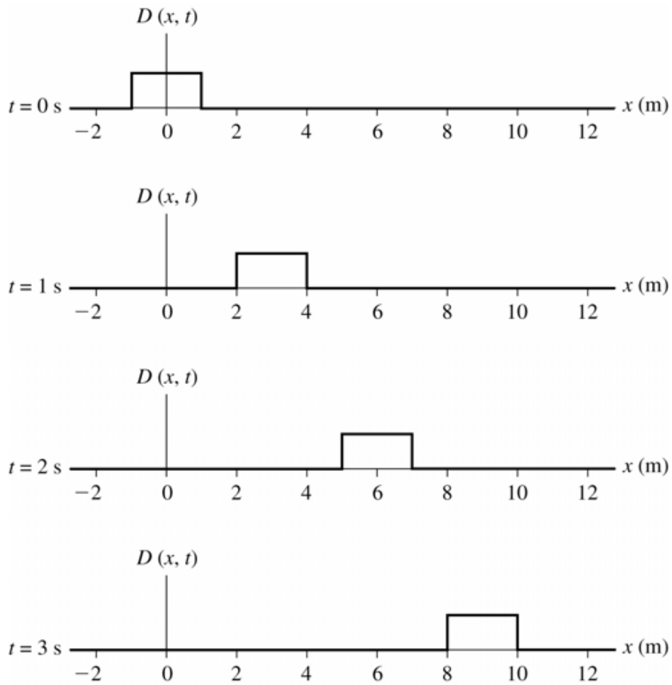
Solutions for Quiz 7 – Week of October 29, 2007

20.40. Visualize: The function $D(x, t)$ represents a pulse that travels in the positive x-direction without changing shape.

Solve: (a)



(b) The leading edge of the pulse moves forward 3 m each second. Thus, the wave speed is 3.0 m/s.

(c) $|x - 3t|$ is a function of the form $D(x - vt)$, so the pulse moves to the right at $v = 3$ m/s.

20.43. Visualize: Please refer to Figure P20.43.

Solve: (a) We see from the history graph that the period $T = 0.20$ s and the wave speed $v = 4.0$ m/s. Thus, the wavelength is

$$\lambda = \frac{v}{f} = vT = (4.0 \text{ m/s})(0.20 \text{ s}) = 0.80 \text{ m}$$

(b) The phase constant ϕ_0 is obtained as follows:

$$D(0 \text{ m}, 0 \text{ s}) = A\sin\phi_0 \Rightarrow -2 \text{ mm} = (2 \text{ mm})\sin\phi_0 \Rightarrow \sin\phi_0 = -1 \Rightarrow \phi_0 = -\tfrac{1}{2}\pi \text{ rad}$$

(c) The displacement equation for the wave is

$$D(x,\ t) = A\sin\left(\frac{2\pi x}{\lambda} - 2\pi ft + \phi_0\right) = (2.0 \text{ mm})\sin\left(\frac{2\pi x}{0.80 \text{ m}} - \frac{2\pi t}{0.20 \text{ s}} - \frac{\pi}{2}\right) = (2.0 \text{ mm})\sin\left(2.5\pi x - 10\pi t - \tfrac{1}{2}\pi\right)$$

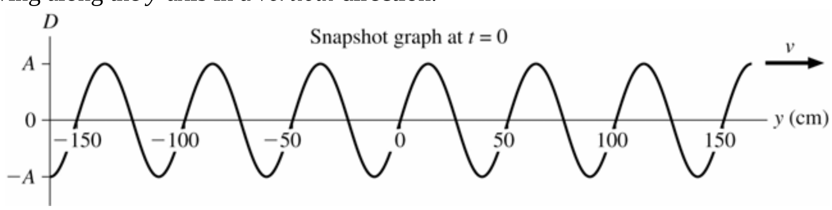
where x and t are in m and s, respectively.

20.44. Solve: The time for the sound wave to travel down the tube and back is $t = 440\ \mu\text{s}$ since 1 division is equal to 100 μs. So, the speed of the sound wave in the liquid is

$$v = \frac{2 \times 25 \text{ cm}}{440\ \mu\text{s}} = 1140 \text{ m/s}$$

20.63. Model: We have a sinusoidal traveling wave going in the +*y*-direction. We will assume a sound speed of 343 m/s.

Solve: The wave's frequency is 686 Hz. Thus the wavelength is $\lambda = v/f = 0.500\text{ m} = 50.0\text{ cm}$. Since one crest is located at y = 12.5 cm at t = 0 s, the other crests will be spaced every 50 cm. The crests will be at 62.5 cm, 112.5 cm, and so on, as well as at −37.5 cm, −87.5 cm, and so on. This is sufficient information to draw the snapshot graph at t = 0 s. Note that the *y*-axis is drawn *horizontally* in this *representation* of the wave, even though the wave is physically moving along the *y*-axis in a *vertical* direction.



(b) The displacement at y = 0 m and t = 0 s is

$$D(0\text{ m},\ 0\text{ s}) = A\sin\phi_0 = 0\text{ m} \Rightarrow \phi_0 = \sin^{-1}(0) = 0 \text{ or } \pi \text{ rad}$$

How do we distinguish between these two mathematically possible answers? Consider the rest of the *y*-axis at t = 0 s. The displacement is

$$D(y,\ t=0) = A\sin\left(\frac{2\pi y}{\lambda} + \phi_0\right) = A\sin\left(\frac{2\pi y}{50\text{ cm}} + \phi_0\right)$$

We know that the displacement at y = 12.5 cm is a crest at t = 0 s ($D = A$), so the sine must equal to 1 at t = 0 s. Using the displacement equation at y = 12.5 cm yields:

$$D(12.5\text{ cm},\ t=0\text{ s}) = A\sin\left(\frac{2\pi(12.5\text{ cm})}{50\text{ cm}} + \phi_0\right) = A\sin\left(\frac{\pi}{2} + \phi_0\right)$$

$\phi_0 = 0$ correctly gives $D = A$, whereas $\phi_0 = \pi$ rad gives $D = -A$. So the phase constant is $\phi_0 = 0$ rad.

(c) Using the known values of λ, f, and ϕ_0, we can write the wave equation as

$$D(y,\ t) = A\sin\left(\frac{2\pi y}{\lambda} - 2\pi ft + \phi_0\right) = A\sin\left[\left(12.57\text{ m}^{-1}\right)y - \left(4310\text{ s}^{-1}\right)t\right]$$

(d) At t = 0 s, one crest is located at y = 12.5 cm and the others are spaced every 50 cm. This leads to the wave front diagram shown below. The wave fronts are numbered.

(e) The period of the wave is

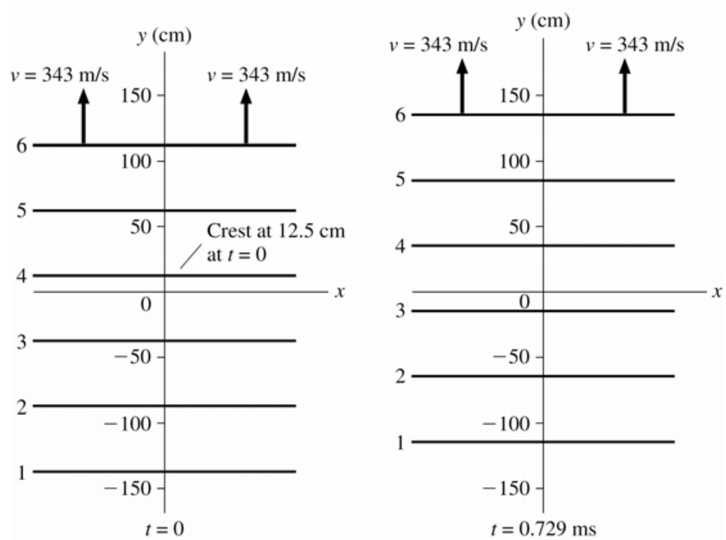
$$T = \frac{1}{f} = \frac{1}{686\text{ Hz}} = 1.458\text{ ms}$$

0.729 ms is exactly $\frac{1}{2}T$ —half a period. During half a period, the wave crests each move forward half a wavelength, or 25 cm.

(f) The *phase* of the wave is the full value of the argument of the sine function at a particular point in space and time. At $y = -12.5$ cm, the phase is

$$\phi = ky - \omega t + \phi_0 = \left(12.57\text{ m}^{-1}\right)y - \left(4310\text{ s}^{-1}\right)t$$
$$= \left(12.57\text{ m}^{-1}\right)\left(-0.125\text{ m}\right) - \left(4310\text{ s}^{-1}\right)\left(0.000729\text{ s}\right) = -4.70\text{ rad} = -\tfrac{3}{2}\pi\text{ rad}$$

This phase corresponds to a wave crest. In the wave front graph at t = 0.729 ms, you can see a crest at y = −12.5 cm. Similarly, the phase at $y = +12.5$ cm is $\phi = -1.57$ rad $= -\frac{1}{2}\pi$ rad. This is a trough.



20.71. Model: We have a traveling wave radiated by the tornado siren.
Solve: **(a)** The power of the source is calculated as follows:

$$I_{50\text{ m}} = 0.10\text{ W/m}^2 = \frac{P_{\text{source}}}{4\pi r^2} = \frac{P_{\text{source}}}{4\pi(50\text{ m})^2} \Rightarrow P_{\text{source}} = \left(0.10\text{ W/m}^2\right)4\pi(50\text{ m})^2 = (1000\pi)\text{ W}$$

The intensity at 1000 m is

$$I_{1000\text{ m}} = \frac{P_{\text{source}}}{4\pi(1000\text{ m})^2} = \frac{(1000\pi)\text{ W}}{4\pi(1000\text{ m})^2} = 250\ \mu\text{W/m}^2$$

(b) The maximum distance is calculated as follows:

$$I = \frac{P_{\text{source}}}{4\pi r^2} \Rightarrow 1.0\times10^{-6}\text{ W/m}^2 = \frac{(1000\pi)\text{ W}}{4\pi r^2} \Rightarrow r = 15.8\text{ km}$$

20.73. Model: The bat's chirping frequency is altered by the Doppler effect. The frequency is increased as the bat approaches and it decreases as the bat recedes away.
Solve: The bat must fly away from you, so that the chirp frequency observed by you is less than 25 kHz. From Equation 20.38,

$$f_- = \frac{f_0}{1 + v_S/v} \Rightarrow 20{,}000\text{ Hz} = \frac{25000\text{ Hz}}{1+\left(\dfrac{v_S}{343\text{ m/s}}\right)} \Rightarrow v_S = 85.8\text{ m/s}$$

Assess: This is a rather large speed: 85.8 m/s ≈ 180 mph. This is not possible for a bat.

20.74. Model: The sound generator's frequency is altered by the Doppler effect. The frequency increases as the generator approaches the student, and it decreases as the generator recedes from the student.
Solve: The generator's speed is

$$v_S = r\omega = r(2\pi f) = (1.0\text{ m})2\pi\left(\frac{100}{60}\text{ rev/s}\right) = 10.47\text{ m/s}$$

The frequency of the approaching generator is

$$f_+ = \frac{f_0}{1 - v_S/v} = \frac{600 \text{ Hz}}{1 - \dfrac{10.47 \text{ m/s}}{343 \text{ m/s}}} = 619 \text{ Hz}$$

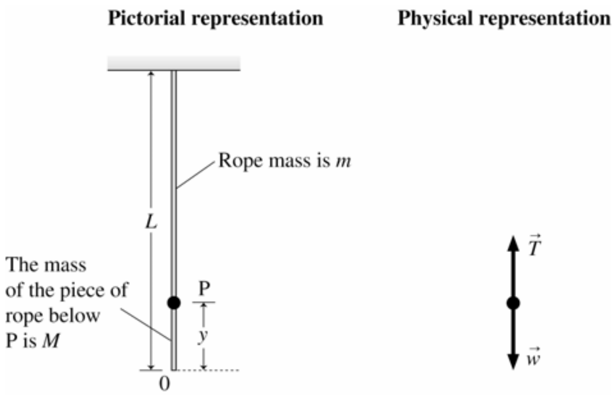
Doppler effect for the receding generator, on the other hand, is

$$f_- = \frac{f_0}{1 + v_S/v} = \frac{600 \text{ Hz}}{1 + \dfrac{10.47 \text{ m/s}}{343 \text{ m/s}}} = 582 \text{ Hz}$$

Thus, the highest and the lowest frequencies heard by the student are 619 Hz and 582 Hz.

20.82. Model: The wave pulse is a traveling wave on a stretched wire.

Visualize:



Solve: (a) At a distance *y* above the lower end of the rope, the point *P* is in static equilibrium. The upward tension in the rope must balance the weight of the rope that hangs below this point. Thus, at this point

$$T = w = Mg = (\mu y)g$$

where $\mu = m/L$ is the linear density of the entire rope. Using Equation 20.2, we get

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu y g}{\mu}} = \sqrt{gy}$$

(b) The time to travel a distance *dy* at *y*, where the wave speed is $v = \sqrt{gy}$, is

$$dt = \frac{dy}{v} = \frac{dy}{\sqrt{gy}}$$

Finding the time for a pulse to travel the length of the rope requires integrating from one end of the rope to the other:

$$\Delta t = \int_0^T dt = \int_0^L \frac{dy}{\sqrt{gy}} = \frac{1}{\sqrt{g}}\left(2\sqrt{y}\Big|_0^L\right) = \frac{2}{\sqrt{g}}\sqrt{L} \Rightarrow \Delta t = 2\sqrt{\frac{L}{g}}$$