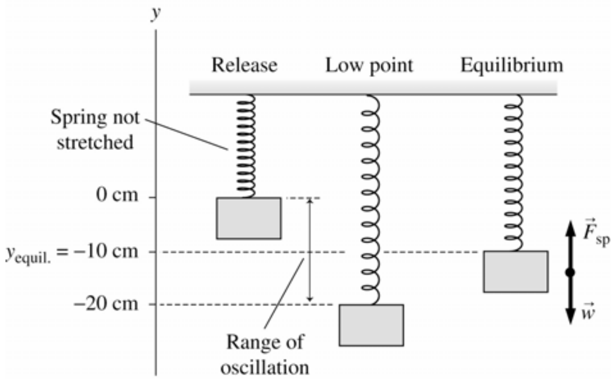
Solutions for Quiz 6 – Week of October 22, 2007

14.50. Model: The mass is in simple harmonic motion.
Visualize:



The high point of the oscillation is *at* the point of release. This conclusion is based on energy conservation. Gravitational potential energy is converted to the spring's elastic potential energy as the mass falls and stretches the spring, then the elastic potential energy is converted 100% back into gravitational potential energy as the mass rises, bringing the mass back to *exactly* its starting height. The total displacement of the oscillation – high point to low point – is 20 cm. Because the oscillations are symmetrical about the equilibrium point, we can deduce that the *equilibrium* point of the spring is 10 cm below the point where the mass is released. The mass oscillates about this equilibrium point with an amplitude of 10 cm, that is, the mass oscillates between 10 cm above and 10 cm below the equilibrium point.

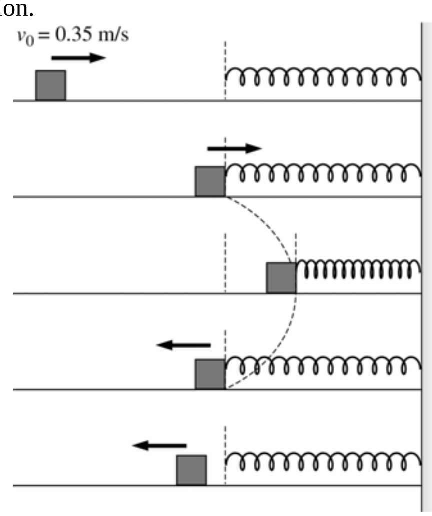
Solve: The equilibrium point is the point where the mass would hang *at rest*, with $F_{sp} = w = mg$. At the equilibrium point, the spring is stretched by $\Delta y = 10\text{ cm} = 0.1\text{ m}$. Hooke's law is $F_{sp} = k\Delta y$, so the equilibrium condition is

$$\left[F_{sp} = k\Delta y\right] = \left[w = mg\right] \Rightarrow \frac{k}{m} = \frac{g}{\Delta y} = \frac{9.8\text{ m/s}^2}{0.1\text{ m}} = 98\text{ s}^{-2}$$

The *ratio* k/m is all we need to find the oscillation frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{98\text{ s}^{-2}} = 1.58\text{ Hz}$$

14.53. Model: Hooke's law for the spring. The spring's compression and decompression constitutes simple harmonic motion.
Visualize:



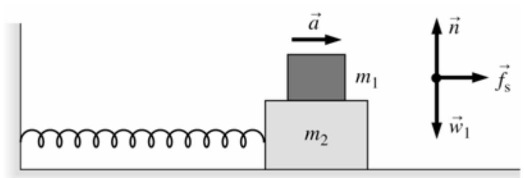
Solve: **(a)** The spring's compression or decompression is one-half of the oscillation cycle. This means the contact time is $\Delta t = \frac{1}{2}T$, where T is the period. The period is calculated as follows:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50 \text{ N/m}}{0.50 \text{ kg}}} = 10 \text{ rad/s} \Rightarrow T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{10 \text{ rad/s}} = 0.628 \text{ s}$$

$$\Rightarrow \Delta t = \frac{T}{2} = 0.314 \text{ s}$$

(b) There is no change in contact time, because period of oscillation is independent of the amplitude or the maximum speed.

14.54. **Model:** The two blocks are in simple harmonic motion, without the upper block slipping. We will also apply the model of static friction between the two blocks.
Visualize:



Solve: The net force acting on the upper block m_1 is the force of friction due to the lower block m_2. The model of static friction gives the maximum force of static friction as

$$f_{\text{s max}} = \mu_s n = \mu_s (m_1 g) = m_1 a_{\text{max}} \Rightarrow a_{\text{max}} = \mu_s g$$

Using $\mu_s = 0.5$, $a_{\text{max}} = \mu_S g = (0.5)(9.8 \text{ m/s}^2) = 4.9 \text{ m/s}^2$. That is, the two blocks will ride together if the maximum acceleration of the system is equal to or less than a_{max}. We can calculate the maximum value of A as follows:

$$a_{\text{max}} = \omega^2 A_{\text{max}} = \frac{k}{m_1 + m_2} A_{\text{max}} \Rightarrow A_{\text{max}} = \frac{a_{\text{max}}(m_1 + m_2)}{k} = \frac{(4.9 \text{ m/s}^2)(1.0 \text{ kg} + 5.0 \text{ kg})}{50 \text{ N/m}} = 0.588 \text{ m}$$

14.58. **Model:** Assume a small angle of oscillation so that the pendulum has simple harmonic motion.
Solve: The time periods of the pendulums on the earth and on Mars are

$$T_{\text{earth}} = 2\pi\sqrt{\frac{L}{g_{\text{earth}}}} \text{ and } T_{\text{Mars}} = 2\pi\sqrt{\frac{L}{g_{\text{Mars}}}}$$

Dividing these two equations,

$$\frac{T_{\text{earth}}}{T_{\text{Mars}}} = \sqrt{\frac{g_{\text{Mars}}}{g_{\text{earth}}}} \Rightarrow g_{\text{Mars}} = g_{\text{earth}}\left(\frac{T_{\text{earth}}}{T_{\text{Mars}}}\right)^2 = (9.8 \text{ m/s}^2)\left(\frac{1.50 \text{ s}}{2.45 \text{ s}}\right)^2 = 3.67 \text{ m/s}^2$$

14.62. **Model:** The block attached to the spring is oscillating in simple harmonic motion.
Solve: **(a)** Because the frequency of an object in simple harmonic motion is independent of the amplitude and/or the maximum velocity, the new frequency is equal to the old frequency of 2.0 Hz.
(b) The speed v_0 of the block just before it is given a blow can be obtained by using the conservation of mechanical energy equation as follows:

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}mv_0^2$$

$$\Rightarrow v_0 = \sqrt{\frac{k}{m}}A = \omega A = (2\pi f)A = (2\pi)(2.0 \text{ Hz})(0.02 \text{ m}) = 0.25 \text{ m/s}$$

The blow to the block provides an impulse that changes the velocity of the block:

$$J_x = F_x \Delta t = \Delta p = mv_f - mv_0$$

$$(-20\ \text{N})(1.0 \times 10^{-3}\ \text{s}) = (0.200\ \text{kg})v_f - (0.200\ \text{kg})(0.25\ \text{m/s}) \Rightarrow v_f = 0.15\ \text{m/s}$$

Since v_f is the new maximum velocity of the block at the equilibrium position, it is equal to $A\omega$. Thus,

$$A = \frac{0.15\ \text{m/s}}{\omega} = \frac{0.15\ \text{m/s}}{2\pi(2.0\ \text{Hz})} = 0.012\ \text{m} = 1.2\ \text{cm}$$

Assess: Because v_f is positive, the block continues to move to the right even after the blow.

14.71. Model: The oscillator is in simple harmonic motion.
Solve: (a) The maximum displacement at time t of a damped oscillator is

$$x_{\text{max}}(t) = Ae^{-t/2\tau} \Rightarrow -\frac{t}{2\tau} = \ln\left(\frac{x_{\text{max}}(t)}{A}\right)$$

Using $x_{\text{max}} = 0.98A$ at $t = 0.5$ s, we can find the time constant τ to be

$$\tau = -\frac{0.5\ \text{s}}{2\ln(0.98)} = 12.375\ \text{s}$$

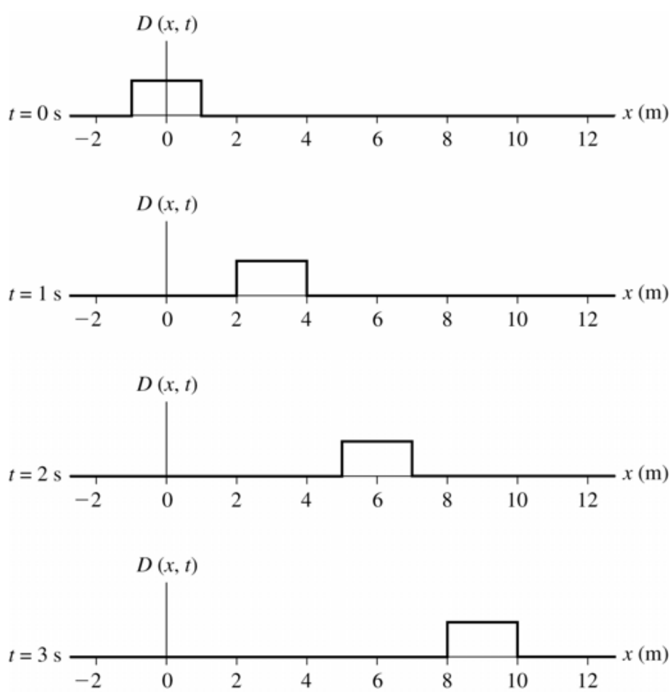
25 oscillations will be completed at $t = 25T = 12.5$ s. At that time, the amplitude will be

$$x_{\text{max, 12.5 s}} = (10.0\ \text{cm})e^{-12.5\ \text{s}/(2)(12.375\ \text{s})} = 6.03\ \text{cm}$$

(b) The energy of a damped oscillator decays more rapidly than the amplitude: $E(t) = E_0 e^{-t/\tau}$. When the energy is 60% of its initial value, $E(t)/E_0 = 0.60$. We can find the time this occurs as follows:

$$-\frac{t}{\tau} = \ln\left(\frac{E(t)}{E_0}\right) \Rightarrow t = -\tau \ln\left(\frac{E(t)}{E_0}\right) = -(12.375\ \text{s})\ln(0.60) = 6.32\ \text{s}$$

20.40. Visualize: The function $D(x, t)$ represents a pulse that travels in the positive x-direction without changing shape.
Solve: (a)



(b) The leading edge of the pulse moves forward 3 m each second. Thus, the wave speed is 3.0 m/s.

(c) $|x - 3t|$ is a function of the form $D(x - vt)$, so the pulse moves to the right at $v = 3$ m/s.

20.43. Visualize: Please refer to Figure P20.43.

Solve: **(a)** We see from the history graph that the period $T = 0.20$ s and the wave speed $v = 4.0$ m/s. Thus, the wavelength is

$$\lambda = \frac{v}{f} = vT = (4.0\text{ m/s})(0.20\text{ s}) = 0.80\text{ m}$$

(b) The phase constant ϕ_0 is obtained as follows:

$$D(0\text{ m}, 0\text{ s}) = A\sin\phi_0 \Rightarrow -2\text{ mm} = (2\text{ mm})\sin\phi_0 \Rightarrow \sin\phi_0 = -1 \Rightarrow \phi_0 = -\tfrac{1}{2}\pi\text{ rad}$$

(c) The displacement equation for the wave is

$$D(x,\ t) = A\sin\left(\frac{2\pi x}{\lambda} - 2\pi ft + \phi_0\right) = (2.0\text{ mm})\sin\left(\frac{2\pi x}{0.80\text{ m}} - \frac{2\pi t}{0.20\text{ s}} - \frac{\pi}{2}\right) = (2.0\text{ mm})\sin\left(2.5\pi x - 10\pi t - \tfrac{1}{2}\pi\right)$$

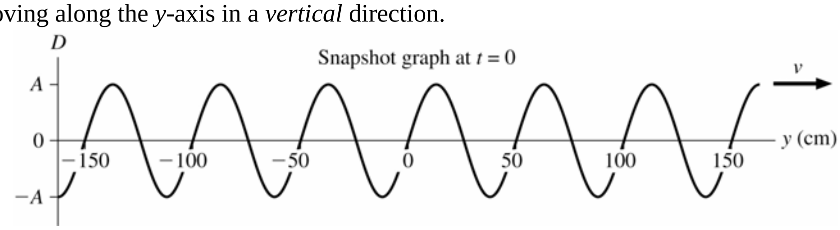
where x and t are in m and s, respectively.

20.44. Solve: The time for the sound wave to travel down the tube and back is $t = 440\ \mu\text{s}$ since 1 division is equal to 100 μs. So, the speed of the sound wave in the liquid is

$$v = \frac{2 \times 25\text{ cm}}{440\ \mu\text{s}} = 1140\text{ m/s}$$

20.63. Model: We have a sinusoidal traveling wave going in the +y-direction. We will assume a sound speed of 343 m/s.

Solve: The wave's frequency is 686 Hz. Thus the wavelength is $\lambda = v/f = 0.500\text{ m} = 50.0\text{ cm}$. Since one crest is located at $y = 12.5$ cm at $t = 0$ s, the other crests will be spaced every 50 cm. The crests will be at 62.5 cm, 112.5 cm, and so on, as well as at –37.5 cm, –87.5 cm, and so on. This is sufficient information to draw the snapshot graph at $t = 0$ s. Note that the y-axis is drawn *horizontally* in this *representation* of the wave, even though the wave is physically moving along the y-axis in a *vertical* direction.



(b) The displacement at $y = 0$ m and $t = 0$ s is

$$D(0\text{ m},\ 0\text{ s}) = A\sin\phi_0 = 0\text{ m} \Rightarrow \phi_0 = \sin^{-1}(0) = 0\text{ or }\pi\text{ rad}$$

How do we distinguish between these two mathematically possible answers? Consider the rest of the y-axis at t = 0 s. The displacement is

$$D(y,\ t = 0) = A\sin\left(\frac{2\pi y}{\lambda} + \phi_0\right) = A\sin\left(\frac{2\pi y}{50\text{ cm}} + \phi_0\right)$$

We know that the displacement at $y = 12.5$ cm is a crest at $t = 0$ s ($D = A$), so the sine must equal to 1 at $t = 0$ s. Using the displacement equation at $y = 12.5$ cm yields:

$$D(12.5\text{ cm}, t = 0\text{ s}) = A\sin\left(\frac{2\pi(12.5\text{ cm})}{50\text{ cm}} + \phi_0\right) = A\sin\left(\frac{\pi}{2} + \phi_0\right)$$

$\phi_0 = 0$ correctly gives $D = A$, whereas $\phi_0 = \pi$ rad gives $D = -A$. So the phase constant is $\phi_0 = 0$ rad.

(c) Using the known values of λ, f, and ϕ_0, we can write the wave equation as

$$D(y,\ t) = A\sin\left(\frac{2\pi y}{\lambda} - 2\pi ft + \phi_0\right) = A\sin\left[\left(12.57\ \text{m}^{-1}\right)y - \left(4310\ \text{s}^{-1}\right)t\right]$$

(d) At $t = 0$ s, one crest is located at $y = 12.5$ cm and the others are spaced every 50 cm. This leads to the wave front diagram shown below. The wave fronts are numbered.

(e) The period of the wave is

$$T = \frac{1}{f} = \frac{1}{686\ \text{Hz}} = 1.458\ \text{ms}$$

0.729 ms is exactly $\frac{1}{2}T$—half a period. During half a period, the wave crests each move forward half a wavelength, or 25 cm.

(f) The *phase* of the wave is the full value of the argument of the sine function at a particular point in space and time. At $y = -12.5$ cm, the phase is

$$\begin{aligned}\phi = ky - \omega t + \phi_0 &= \left(12.57\ \text{m}^{-1}\right)y - \left(4310\ \text{s}^{-1}\right)t \\ &= \left(12.57\ \text{m}^{-1}\right)\left(-0.125\ \text{m}\right) - \left(4310\ \text{s}^{-1}\right)\left(0.000729\ \text{s}\right) = -4.70\ \text{rad} = -\tfrac{3}{2}\pi\ \text{rad}\end{aligned}$$

This phase corresponds to a wave crest. In the wave front graph at $t = 0.729$ ms, you can see a crest at $y = -12.5$ cm. Similarly, the phase at $y = +12.5$ cm is $\phi = -1.57$ rad $= -\frac{1}{2}\pi$ rad. This is a trough.