Solutions for Quiz 5 – Week of October 15, 2007

14.37. **Model:** The block attached to the spring is in simple harmonic motion.
Visualize: The position and the velocity of the block are given by the equations

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0)$$

Solve: To graph $x(t)$ we need to determine ω, ϕ_0, and A. These quantities will be found by using the initial ($t = 0$ s) conditions on $x(t)$ and $v_x(t)$. The period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.0\text{ kg}}{20\text{ N/m}}} = 1.405\text{ s} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi\text{ rad}}{1.405\text{ s}} = 4.472\text{ rad/s}$$

At $t = 0$ s, $x_0 = A\cos\phi_0$ and $v_{0x} = -A\omega\sin\phi_0$. Dividing these equations,

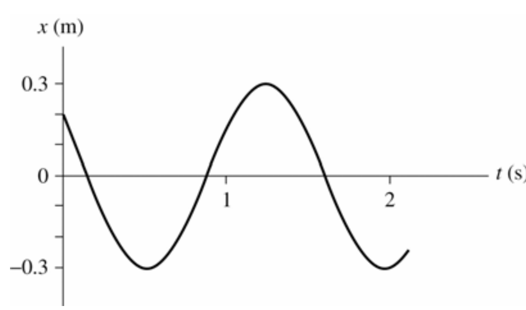
$$\tan\phi_0 = -\frac{v_{0x}}{\omega x_0} = -\frac{(-1.0\text{ m/s})}{(4.472\text{ rad/s})(0.20\text{ m})} = 1.1181 \Rightarrow \phi_0 = 0.841\text{ rad}$$

From the initial conditions,

$$A = \sqrt{x_0^2 + \left(\frac{v_{0x}}{\omega}\right)^2} = \sqrt{(0.20\text{ m})^2 + \left(\frac{-1.0\text{ m/s}}{4.472\text{ rad/s}}\right)^2} = 0.300\text{ m}$$

The position-versus-time graph can now be plotted using the equation

$$x(t) = (0.300\text{ m})\cos\left[(4.472\text{ rad/s})t + 0.841\text{ rad}\right]$$



14.38. **Solve:** The object's position as a function of time is $x(t) = A\cos(\omega t + \phi_0)$. Letting $x = 0$ m at $t = 0$ s, gives

$$0 = A\cos\phi_0 \Rightarrow \phi_0 = \pm\tfrac{1}{2}\pi$$

Since the object is traveling to the right, it is in the lower half of the circular motion diagram, giving a phase constant between $-\pi$ and 0 radians. Thus, $\phi_0 = -\frac{1}{2}\pi$ and

$$x(t) = A\cos(\omega t - \tfrac{1}{2}\pi) \Rightarrow x(t) = A\sin\omega t = (0.10\text{ m})\sin(\tfrac{1}{2}\pi t)$$

where we have used $A = 0.10$ m and

$$\omega = \frac{2\pi}{T} = \frac{2\pi\text{ rad}}{4.0\text{ s}} = \frac{\pi}{2}\text{ rad/s}$$

Let us now find t where $x = 0.060$ m:

$$0.060\text{ m} = (0.10\text{ m})\sin\left(\frac{\pi}{2}t\right) \Rightarrow t = \frac{2}{\pi}\sin^{-1}\left(\frac{0.060\text{ m}}{0.10\text{ m}}\right) = 0.410\text{ s}$$

Assess: The answer is reasonable because it is approximately $\frac{1}{8}$ of the period.

14.43. Model: The ball attached to a spring is in simple harmonic motion.
Solve: **(a)** Let $t = 0$ s be the instant when $x_0 = -5$ cm and $v_0 = 20$ cm/s. The oscillation frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.5\ \text{N/m}}{0.10\ \text{kg}}} = 5.0\ \text{rad/s}$$

Using Equation 14.27, the amplitude of the oscillation is

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(-5\ \text{cm})^2 + \left(\frac{20\ \text{cm/s}}{5\ \text{rad/s}}\right)^2} = 6.40\ \text{cm}$$

(b) The maximum acceleration is $a_{\text{max}} = \omega^2 A = 160\ \text{cm/s}^2$.

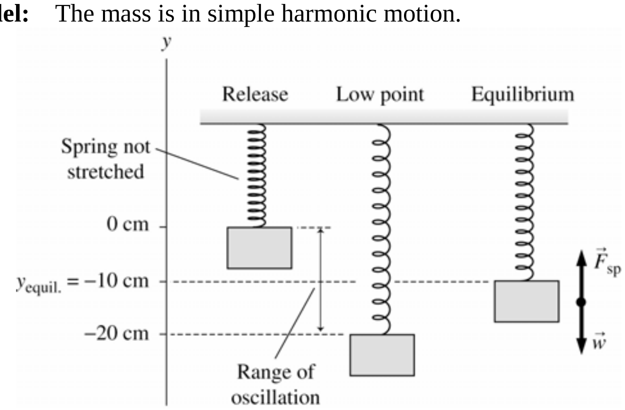
(c) For an oscillator, the acceleration is most positive $(a = a_{\text{max}})$ when the displacement is most negative $(x = -x_{\text{max}} = -A)$. So the acceleration is maximum when $x = -6.40$ cm.

(d) We can use the conservation of energy between $x_0 = -5$ cm and $x_1 = 3$ cm:

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 \Rightarrow v_1 = \sqrt{v_0^2 + \frac{k}{m}\left(x_0^2 - x_1^2\right)} = 0.283\ \text{m/s} = 28.3\ \text{cm/s}$$

Because k is known in SI units of N/m, the energy calculation *must* be done using SI units of m, m/s, and kg.

14.50. Model: The mass is in simple harmonic motion.
Visualize:



The high point of the oscillation is *at* the point of release. This conclusion is based on energy conservation. Gravitational potential energy is converted to the spring's elastic potential energy as the mass falls and stretches the spring, then the elastic potential energy is converted 100% back into gravitational potential energy as the mass rises, bringing the mass back to *exactly* its starting height. The total displacement of the oscillation – high point to low point – is 20 cm. Because the oscillations are symmetrical about the equilibrium point, we can deduce that the *equilibrium* point of the spring is 10 cm below the point where the mass is released. The mass oscillates about this equilibrium point with an amplitude of 10 cm, that is, the mass oscillates between 10 cm above and 10 cm below the equilibrium point.

Solve: The equilibrium point is the point where the mass would hang *at rest*, with $F_{\text{sp}} = w = mg$. At the equilibrium point, the spring is stretched by $\Delta y = 10$ cm $= 0.1$ m. Hooke's law is $F_{\text{sp}} = k\Delta y$, so the equilibrium condition is

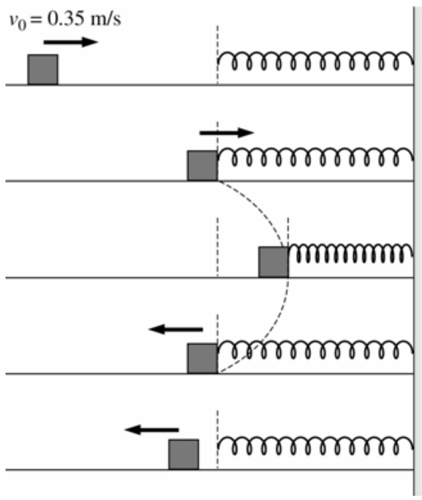
$$\left[F_{\text{sp}} = k\Delta y\right] = \left[w = mg\right] \Rightarrow \frac{k}{m} = \frac{g}{\Delta y} = \frac{9.8\ \text{m/s}^2}{0.1\ \text{m}} = 98\ \text{s}^{-2}$$

The *ratio* k/m is all we need to find the oscillation frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{98\ \text{s}^{-2}} = 1.58\ \text{Hz}$$

14.53. Model: Hooke's law for the spring. The spring's compression and decompression constitutes simple harmonic motion.

Visualize:



Solve: **(a)** The spring's compression or decompression is one-half of the oscillation cycle. This means the contact time is $\Delta t = \frac{1}{2}T$, where T is the period. The period is calculated as follows:

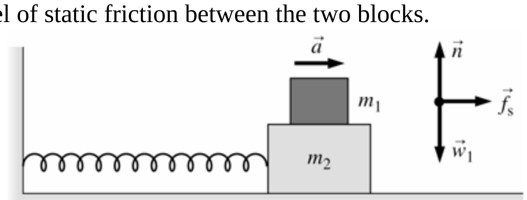
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\ \text{N/m}}{0.50\ \text{kg}}} = 10\ \text{rad/s} \Rightarrow T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{10\ \text{rad/s}} = 0.628\ \text{s}$$

$$\Rightarrow \Delta t = \frac{T}{2} = 0.314\ \text{s}$$

(b) There is no change in contact time, because period of oscillation is independent of the amplitude or the maximum speed.

14.54. Model: The two blocks are in simple harmonic motion, without the upper block slipping. We will also apply the model of static friction between the two blocks.

Visualize:



Solve: The net force acting on the upper block m_1 is the force of friction due to the lower block m_2. The model of static friction gives the maximum force of static friction as

$$f_{\text{s max}} = \mu_s n = \mu_s (m_1 g) = m_1 a_{\text{max}} \Rightarrow a_{\text{max}} = \mu_s g$$

Using $\mu_s = 0.5$, $a_{\text{max}} = \mu_s g = (0.5)(9.8\ \text{m/s}^2) = 4.9\ \text{m/s}^2$. That is, the two blocks will ride together if the maximum acceleration of the system is equal to or less than a_{max}. We can calculate the maximum value of A as follows:

$$a_{\text{max}} = \omega^2 A_{\text{max}} = \frac{k}{m_1 + m_2} A_{\text{max}} \Rightarrow A_{\text{max}} = \frac{a_{\text{max}}(m_1 + m_2)}{k} = \frac{(4.9\ \text{m/s}^2)(1.0\ \text{kg} + 5.0\ \text{kg})}{50\ \text{N/m}} = 0.588\ \text{m}$$

14.58. **Model:** Assume a small angle of oscillation so that the pendulum has simple harmonic motion.
Solve: The time periods of the pendulums on the earth and on Mars are

$$T_{\text{earth}} = 2\pi\sqrt{\frac{L}{g_{\text{earth}}}} \text{ and } T_{\text{Mars}} = 2\pi\sqrt{\frac{L}{g_{\text{Mars}}}}$$

Dividing these two equations,

$$\frac{T_{\text{earth}}}{T_{\text{Mars}}} = \sqrt{\frac{g_{\text{Mars}}}{g_{\text{earth}}}} \Rightarrow g_{\text{Mars}} = g_{\text{earth}}\left(\frac{T_{\text{earth}}}{T_{\text{Mars}}}\right)^2 = \left(9.8 \text{ m/s}^2\right)\left(\frac{1.50 \text{ s}}{2.45 \text{ s}}\right)^2 = 3.67 \text{ m/s}^2$$

14.62. **Model:** The block attached to the spring is oscillating in simple harmonic motion.
Solve: **(a)** Because the frequency of an object in simple harmonic motion is independent of the amplitude and/or the maximum velocity, the new frequency is equal to the old frequency of 2.0 Hz.
(b) The speed v_0 of the block just before it is given a blow can be obtained by using the conservation of mechanical energy equation as follows:

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}mv_0^2$$

$$\Rightarrow v_0 = \sqrt{\frac{k}{m}}A = \omega A = \left(2\pi f\right)A = \left(2\pi\right)\left(2.0 \text{ Hz}\right)\left(0.02 \text{ m}\right) = 0.25 \text{ m/s}$$

The blow to the block provides an impulse that changes the velocity of the block:

$$J_x = F_x\Delta t = \Delta p = mv_{\text{f}} - mv_0$$

$$\left(-20 \text{ N}\right)\left(1.0\times10^{-3} \text{ s}\right) = \left(0.200 \text{ kg}\right)v_{\text{f}} - \left(0.200 \text{ kg}\right)\left(0.25 \text{ m/s}\right) \Rightarrow v_{\text{f}} = 0.15 \text{ m/s}$$

Since v_{f} is the new maximum velocity of the block at the equilibrium position, it is equal to $A\omega$. Thus,

$$A = \frac{0.15 \text{ m/s}}{\omega} = \frac{0.15 \text{ m/s}}{2\pi\left(2.0 \text{ Hz}\right)} = 0.012 \text{ m} = 1.2 \text{ cm}$$

Assess: Because v_{f} is positive, the block continues to move to the right even after the blow.

14.71. **Model:** The oscillator is in simple harmonic motion.
Solve: **(a)** The maximum displacement at time t of a damped oscillator is

$$x_{\text{max}}\left(t\right) = Ae^{-t/2\tau} \Rightarrow -\frac{t}{2\tau} = \ln\left(\frac{x_{\text{max}}\left(t\right)}{A}\right)$$

Using $x_{\text{max}} = 0.98\text{A}$ at $t = 0.5$ s, we can find the time constant τ to be

$$\tau = -\frac{0.5 \text{ s}}{2\ln\left(0.98\right)} = 12.375 \text{ s}$$

25 oscillations will be completed at $t = 25T = 12.5$ s. At that time, the amplitude will be

$$x_{\text{max, 12.5 s}} = \left(10.0 \text{ cm}\right)e^{-12.5 \text{ s}/(2)(12.375 \text{ s})} = 6.03 \text{ cm}$$

(b) The energy of a damped oscillator decays more rapidly than the amplitude: $E(t) = E_0e^{-t/\tau}$. When the energy is 60% of its initial value, $E(t)/E_0 = 0.60$. We can find the time this occurs as follows:

$$-\frac{t}{\tau} = \ln\left(\frac{E\left(t\right)}{E_0}\right) \Rightarrow t = -\tau\ln\left(\frac{E\left(t\right)}{E_0}\right) = -\left(12.375 \text{ s}\right)\ln\left(0.60\right) = 6.32 \text{ s}$$