Solutions for Quiz 4 – Week of October 8, 2007

14.32. **Model:** The object is undergoing simple harmonic motion.

Solve: The velocity of an object oscillating on a spring is

$$v_x(t) = -A\omega\sin(\omega t + \phi_0)$$

(a) For $A' = 2A$ and $\omega' = \omega/2$, we have

$$v'_x(t) = -(2A)(\omega/2)\sin\left[(\omega/2)t + \phi_0\right] = -A\omega\sin(\omega t/2 + \phi_0)$$

That is, the maximum velocity $A'\omega'$ remains the same, but the frequency of oscillation is halved.
(b) For $m' = 4m$,

$$\omega' = \sqrt{\frac{k}{m'}} = \sqrt{\frac{k}{4m}} = \frac{\omega}{2} \text{ and } A' = A \Rightarrow v'_{\text{max}} = A'\omega' = A\omega/2 = v_{\text{max}}/2$$

Quadrupling of the mass halves the frequency or doubles the time period, and halves the maximum velocity.

14.33. **Visualize:** Please refer to Figure P14.33.
Solve: The position and the velocity of a particle in simple harmonic motion are

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0) = -v_{\text{max}}\sin(\omega t + \phi_0)$$

(a) At $t = 0$ s, the equation for x yields

$$(5.0\text{ cm}) = (10.0\text{ cm})\cos(\phi_0) \Rightarrow \phi_0 = \cos^{-1}(0.5) = \pm\tfrac{1}{3}\pi \text{ rad}$$

Because the particle is moving to the right at $t = 0$ s, it is in the lower half of the circular motion diagram, and the phase constant is between π and 2π radians. Thus, $\phi_0 = -\tfrac{1}{3}\pi$ rad.
(b) At $t = 0$ s,

$$v_{0x} = -A\omega\sin\phi_0 = -(10.0\text{ cm})\left(\frac{2\pi}{T}\right)\sin\left(-\frac{\pi}{3}\right) = +6.80\text{ cm}$$

(c) The maximum speed is

$$v_{\text{max}} = \omega A = \left(\frac{2\pi}{8.0\text{ s}}\right)(10.0\text{ cm}) = 7.85\text{ cm/s}$$

Assess: The positive velocity at $t = 0$ s is consistent with the position-versus-time graph and the negative sign of the phase constant.

14.35. **Model:** The vertical mass/spring systems are in simple harmonic motion.
Visualize: Please refer to Figure P14.35.
Solve: **(a)** For system A, the maximum speed while traveling in the upward direction corresponds to the maximum positive slope, which is at $t = 3.0$ s. The frequency of oscillation is 0.25 Hz.
(b) For system B, all the energy is potential energy when the position is at maximum amplitude, which for the first time is at $t = 1.5$ s. The time period of system B is thus 6.0 s.
(c) Spring/mass A undergoes three oscillations in 12 s, giving it a period $T_A = 4.0$ s. Spring/mass B undergoes 2 oscillations in 12 s, giving it a period $T_B = 6.0$ s. We have

$$T_A = 2\pi\sqrt{\frac{m_A}{k_A}} \text{ and } T_B = 2\pi\sqrt{\frac{m_B}{k_B}} \Rightarrow \frac{T_A}{T_B} = \sqrt{\left(\frac{m_A}{m_B}\right)\left(\frac{k_B}{k_A}\right)} = \frac{4.0\text{ s}}{6.0\text{ s}} = \frac{2}{3}$$

If $m_A = m_B$, then

$$\frac{k_B}{k_A} = \frac{4}{9} \Rightarrow \frac{k_A}{k_B} = \frac{9}{4} = 2.25$$

14.37. Model: The block attached to the spring is in simple harmonic motion.
Visualize: The position and the velocity of the block are given by the equations

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0)$$

Solve: To graph $x(t)$ we need to determine ω, ϕ_0, and A. These quantities will be found by using the initial ($t = 0$ s) conditions on $x(t)$ and $v_x(t)$. The period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.0 \text{ kg}}{20 \text{ N/m}}} = 1.405 \text{ s} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{1.405 \text{ s}} = 4.472 \text{ rad/s}$$

At $t = 0$ s, $x_0 = A\cos\phi_0$ and $v_{0x} = -A\omega\sin\phi_0$. Dividing these equations,

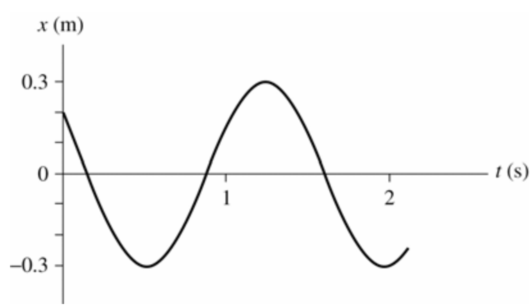
$$\tan\phi_0 = -\frac{v_{0x}}{\omega x_0} = -\frac{(-1.0 \text{ m/s})}{(4.472 \text{ rad/s})(0.20 \text{ m})} = 1.1181 \Rightarrow \phi_0 = 0.841 \text{ rad}$$

From the initial conditions,

$$A = \sqrt{x_0^2 + \left(\frac{v_{0x}}{\omega}\right)^2} = \sqrt{(0.20 \text{ m})^2 + \left(\frac{-1.0 \text{ m/s}}{4.472 \text{ rad/s}}\right)^2} = 0.300 \text{ m}$$

The position-versus-time graph can now be plotted using the equation

$$x(t) = (0.300 \text{ m})\cos\left[(4.472 \text{ rad/s})t + 0.841 \text{ rad}\right]$$



14.38. Solve: The object's position as a function of time is $x(t) = A\cos(\omega t + \phi_0)$. Letting $x = 0$ m at $t = 0$ s, gives

$$0 = A\cos\phi_0 \Rightarrow \phi_0 = \pm\tfrac{1}{2}\pi$$

Since the object is traveling to the right, it is in the lower half of the circular motion diagram, giving a phase constant between $-\pi$ and 0 radians. Thus, $\phi_0 = -\tfrac{1}{2}\pi$ and

$$x(t) = A\cos(\omega t - \tfrac{1}{2}\pi) \Rightarrow x(t) = A\sin\omega t = (0.10 \text{ m})\sin(\tfrac{1}{2}\pi t)$$

where we have used $A = 0.10$ m and

$$\omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{4.0 \text{ s}} = \frac{\pi}{2} \text{ rad/s}$$

Let us now find t where $x = 0.060$ m:

$$0.060\text{ m} = (0.10\text{ m})\sin\left(\frac{\pi}{2}t\right) \Rightarrow t = \frac{2}{\pi}\sin^{-1}\left(\frac{0.060\text{ m}}{0.10\text{ m}}\right) = 0.410\text{ s}$$

Assess: The answer is reasonable because it is approximately $\frac{1}{8}$ of the period.

14.43. Model: The ball attached to a spring is in simple harmonic motion.
Solve: **(a)** Let $t = 0$ s be the instant when $x_0 = -5$ cm and $v_0 = 20$ cm/s. The oscillation frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.5\text{ N/m}}{0.10\text{ kg}}} = 5.0\text{ rad/s}$$

Using Equation 14.27, the amplitude of the oscillation is

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(-5\text{ cm})^2 + \left(\frac{20\text{ cm/s}}{5\text{ rad/s}}\right)^2} = 6.40\text{ cm}$$

(b) The maximum acceleration is $a_{\max} = \omega^2 A = 160\text{ cm/s}^2$.

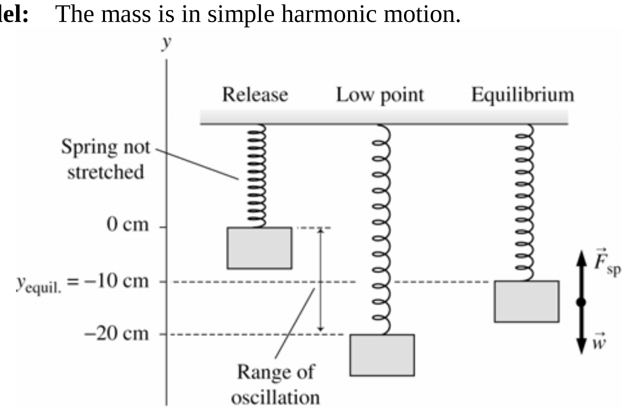
(c) For an oscillator, the acceleration is most positive $(a = a_{\max})$ when the displacement is most negative $(x = -x_{\max} = -A)$. So the acceleration is maximum when $x = -6.40$ cm.

(d) We can use the conservation of energy between $x_0 = -5$ cm and $x_1 = 3$ cm:

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 \Rightarrow v_1 = \sqrt{v_0^2 + \frac{k}{m}\left(x_0^2 - x_1^2\right)} = 0.283\text{ m/s} = 28.3\text{ cm/s}$$

Because k is known in SI units of N/m, the energy calculation *must* be done using SI units of m, m/s, and kg.

14.50. Model: The mass is in simple harmonic motion.
Visualize:



The high point of the oscillation is *at* the point of release. This conclusion is based on energy conservation. Gravitational potential energy is converted to the spring's elastic potential energy as the mass falls and stretches the spring, then the elastic potential energy is converted 100% back into gravitational potential energy as the mass rises, bringing the mass back to *exactly* its starting height. The total displacement of the oscillation – high point to low point – is 20 cm. Because the oscillations are symmetrical about the equilibrium point, we can deduce that the *equilibrium* point of the spring is 10 cm below the point where the mass is released. The mass oscillates about this equilibrium point with an amplitude of 10 cm, that is, the mass oscillates between 10 cm above and 10 cm below the equilibrium point.

Solve: The equilibrium point is the point where the mass would hang *at rest*, with $F_{sp} = w = mg$. At the equilibrium point, the spring is stretched by $\Delta y = 10$ cm $= 0.1$ m. Hooke's law is $F_{sp} = k\Delta y$, so the equilibrium condition is

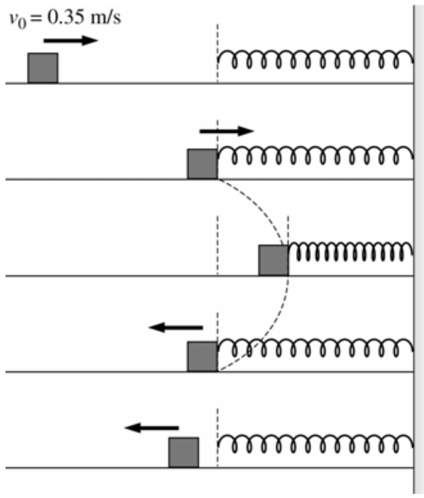
$$\left[F_{sp} = k\Delta y\right] = \left[w = mg\right] \Rightarrow \frac{k}{m} = \frac{g}{\Delta y} = \frac{9.8\text{ m/s}^2}{0.1\text{ m}} = 98\text{ s}^{-2}$$

The *ratio* k/m is all we need to find the oscillation frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{98\ \text{s}^{-2}} = 1.58\ \text{Hz}$$

14.53. Model: Hooke's law for the spring. The spring's compression and decompression constitutes simple harmonic motion.

Visualize:



Solve: **(a)** The spring's compression or decompression is one-half of the oscillation cycle. This means the contact time is $\Delta t = \frac{1}{2}T$, where T is the period. The period is calculated as follows:

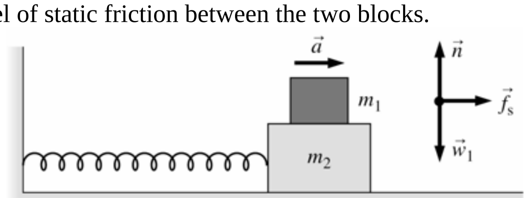
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\ \text{N/m}}{0.50\ \text{kg}}} = 10\ \text{rad/s} \Rightarrow T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{10\ \text{rad/s}} = 0.628\ \text{s}$$

$$\Rightarrow \Delta t = \frac{T}{2} = 0.314\ \text{s}$$

(b) There is no change in contact time, because period of oscillation is independent of the amplitude or the maximum speed.

14.54. Model: The two blocks are in simple harmonic motion, without the upper block slipping. We will also apply the model of static friction between the two blocks.

Visualize:



Solve: The net force acting on the upper block m_1 is the force of friction due to the lower block m_2. The model of static friction gives the maximum force of static friction as

$$f_{\text{s max}} = \mu_s n = \mu_s (m_1 g) = m_1 a_{\max} \Rightarrow a_{\max} = \mu_s g$$

Using $\mu_s = 0.5$, $a_{\max} = \mu_s g = (0.5)(9.8\ \text{m/s}^2) = 4.9\ \text{m/s}^2$. That is, the two blocks will ride together if the maximum acceleration of the system is equal to or less than $a_{\max}$. We can calculate the maximum value of A as follows:

$$a_{\max} = \omega^2 A_{\max} = \frac{k}{m_1 + m_2} A_{\max} \Rightarrow A_{\max} = \frac{a_{\max}(m_1 + m_2)}{k} = \frac{(4.9\ \text{m/s}^2)(1.0\ \text{kg} + 5.0\ \text{kg})}{50\ \text{N/m}} = 0.588\ \text{m}$$