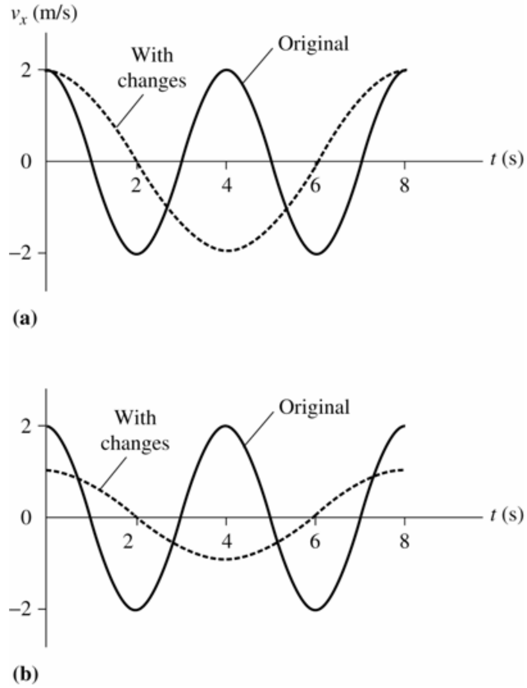
Solutions for Quiz 3 – Week of October 1, 2007

14.32. **Model:** The object is undergoing simple harmonic motion.

Visualize:



Solve: The velocity of an object oscillating on a spring is

$$v_x(t) = -A\omega\sin(\omega t + \phi_0)$$

(a) For $A' = 2A$ and $\omega' = \omega/2$, we have

$$v_x'(t) = -(2A)(\omega/2)\sin\left[(\omega/2)t + \phi_0\right] = -A\omega\sin(\omega t/2 + \phi_0)$$

That is, the maximum velocity $A'\omega'$ remains the same, but the frequency of oscillation is halved.

(b) For $m' = 4m$,

$$\omega' = \sqrt{\frac{k}{m'}} = \sqrt{\frac{k}{4m}} = \frac{\omega}{2} \text{ and } A' = A \Rightarrow v'_{max} = A'\omega' = A\omega/2 = v_{max}/2$$

Quadrupling of the mass halves the frequency or doubles the time period, and halves the maximum velocity.

14.33. **Visualize:** Please refer to Figure P14.33.

Solve: The position and the velocity of a particle in simple harmonic motion are

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0) = -v_{max}\sin(\omega t + \phi_0)$$

(a) At $t = 0$ s, the equation for x yields

$$(5.0 \text{ cm}) = (10.0 \text{ cm})\cos(\phi_0) \Rightarrow \phi_0 = \cos^{-1}(0.5) = \pm\tfrac{1}{3}\pi \text{ rad}$$

Because the particle is moving to the right at $t = 0$ s, it is in the lower half of the circular motion diagram, and the phase constant is between π and 2π radians. Thus, $\phi_0 = -\tfrac{1}{3}\pi$ rad.

(b) At $t = 0$ s,

$$v_{0x} = -A\omega\sin\phi_0 = -(10.0 \text{ cm})\left(\frac{2\pi}{T}\right)\sin\left(-\frac{\pi}{3}\right) = +6.80 \text{ cm}$$

(c) The maximum speed is

$$v_{\text{max}} = \omega A = \left(\frac{2\pi}{8.0 \text{ s}}\right)(10.0 \text{ cm}) = 7.85 \text{ cm/s}$$

Assess: The positive velocity at $t = 0$ s is consistent with the position-versus-time graph and the negative sign of the phase constant.

14.35. Model: The vertical mass/spring systems are in simple harmonic motion.
Visualize: Please refer to Figure P14.35.
Solve: (a) For system A, the maximum speed while traveling in the upward direction corresponds to the maximum positive slope, which is at $t = 3.0$ s. The frequency of oscillation is 0.25 Hz.
(b) For system B, all the energy is potential energy when the position is at maximum amplitude, which for the first time is at $t = 1.5$ s. The time period of system B is thus 6.0 s.
(c) Spring/mass A undergoes three oscillations in 12 s, giving it a period $T_A = 4.0$ s. Spring/mass B undergoes 2 oscillations in 12 s, giving it a period $T_B = 6.0$ s. We have

$$T_A = 2\pi\sqrt{\frac{m_A}{k_A}} \text{ and } T_B = 2\pi\sqrt{\frac{m_B}{k_B}} \Rightarrow \frac{T_A}{T_B} = \sqrt{\left(\frac{m_A}{m_B}\right)\left(\frac{k_B}{k_A}\right)} = \frac{4.0 \text{ s}}{6.0 \text{ s}} = \frac{2}{3}$$

If $m_A = m_B$, then

$$\frac{k_B}{k_A} = \frac{4}{9} \Rightarrow \frac{k_A}{k_B} = \frac{9}{4} = 2.25$$

14.37. Model: The block attached to the spring is in simple harmonic motion.
Visualize: The position and the velocity of the block are given by the equations

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0)$$

Solve: To graph $x(t)$ we need to determine ω, ϕ_0, and A. These quantities will be found by using the initial ($t = 0$ s) conditions on $x(t)$ and $v_x(t)$. The period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.0 \text{ kg}}{20 \text{ N/m}}} = 1.405 \text{ s} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{1.405 \text{ s}} = 4.472 \text{ rad/s}$$

At $t = 0$ s, $x_0 = A\cos\phi_0$ and $v_{0x} = -A\omega\sin\phi_0$. Dividing these equations,

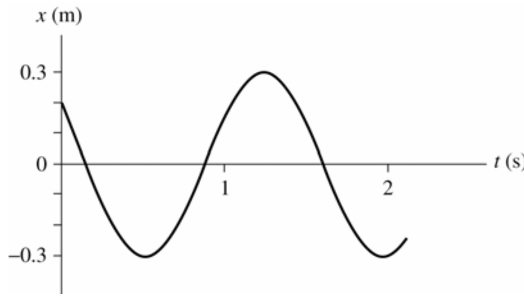
$$\tan\phi_0 = -\frac{v_{0x}}{\omega x_0} = -\frac{(-1.0 \text{ m/s})}{(4.472 \text{ rad/s})(0.20 \text{ m})} = 1.1181 \Rightarrow \phi_0 = 0.841 \text{ rad}$$

From the initial conditions,

$$A = \sqrt{x_0^2 + \left(\frac{v_{0x}}{\omega}\right)^2} = \sqrt{(0.20 \text{ m})^2 + \left(\frac{-1.0 \text{ m/s}}{4.472 \text{ rad/s}}\right)^2} = 0.300 \text{ m}$$

The position-versus-time graph can now be plotted using the equation

$$x(t) = (0.300 \text{ m})\cos[(4.472 \text{ rad/s})t + 0.841 \text{ rad}]$$



14.38. Solve: The object's position as a function of time is $x(t) = A\cos(\omega t + \phi_0)$. Letting $x = 0$ m at $t = 0$ s, gives

$$0 = A\cos\phi_0 \Rightarrow \phi_0 = \pm\tfrac{1}{2}\pi$$

Since the object is traveling to the right, it is in the lower half of the circular motion diagram, giving a phase constant between $-\pi$ and 0 radians. Thus, $\phi_0 = -\tfrac{1}{2}\pi$ and

$$x(t) = A\cos(\omega t - \tfrac{1}{2}\pi) \Rightarrow x(t) = A\sin\omega t = (0.10\text{ m})\sin(\tfrac{1}{2}\pi t)$$

where we have used $A = 0.10$ m and

$$\omega = \frac{2\pi}{T} = \frac{2\pi\text{ rad}}{4.0\text{ s}} = \frac{\pi}{2}\text{ rad/s}$$

Let us now find t where $x = 0.060$ m:

$$0.060\text{ m} = (0.10\text{ m})\sin\left(\frac{\pi}{2}t\right) \Rightarrow t = \frac{2}{\pi}\sin^{-1}\left(\frac{0.060\text{ m}}{0.10\text{ m}}\right) = 0.410\text{ s}$$

Assess: The answer is reasonable because it is approximately $\frac{1}{8}$ of the period.

14.43. Model: The ball attached to a spring is in simple harmonic motion.

Solve: **(a)** Let $t = 0$ s be the instant when $x_0 = -5$ cm and $v_0 = 20$ cm/s. The oscillation frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.5\text{ N/m}}{0.10\text{ kg}}} = 5.0\text{ rad/s}$$

Using Equation 14.27, the amplitude of the oscillation is

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(-5\text{ cm})^2 + \left(\frac{20\text{ cm/s}}{5\text{ rad/s}}\right)^2} = 6.40\text{ cm}$$

(b) The maximum acceleration is $a_{max} = \omega^2 A = 160\text{ cm/s}^2$.

(c) For an oscillator, the acceleration is most positive $(a = a_{max})$ when the displacement is most negative $(x = -x_{max} = -A)$. So the acceleration is maximum when $x = -6.40$ cm.

(d) We can use the conservation of energy between $x_0 = -5$ cm and $x_1 = 3$ cm:

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 \Rightarrow v_1 = \sqrt{v_0^2 + \frac{k}{m}(x_0^2 - x_1^2)} = 0.283\text{ m/s} = 28.3\text{ cm/s}$$

Because k is known in SI units of N/m, the energy calculation *must* be done using SI units of m, m/s, and kg.