Solutions for Quiz 10 – Week of November 19, 2007

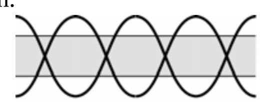
21.48. Model: A tube forms standing waves.

Solve: (a) The fundamental frequency cannot be 390 Hz because 520 Hz and 650 Hz are not integer multiples of it. But we note that the *difference* between 390 Hz and 520 Hz is 130 Hz as is the *difference* between 520 Hz and 650 Hz. We see that 390 Hz = 3 × 130 Hz = $3f_1$, 520 Hz = $4f_1$, and 650 Hz = $5f_1$. So we are seeing the third, fourth, and fifth harmonics of a tube whose fundamental frequency is 130 Hz. According to Equation 21.17, this is an open-open tube because $f_m = mf_1$ with $m = 1, 2, 3, 4, \ldots$ For an open-closed tube m has only odd values.

(b) Knowing f_1, we can now find the length of the tube:

$$L = \frac{v}{2f_1} = \frac{343 \text{ m/s}}{2(130 \text{ Hz})} = 1.32 \text{ m}$$

(c) 520 Hz is the fourth harmonic. This is a sound wave, not a wave on a string, so the wave will have four nodes and will have antinodes at the ends, as shown.



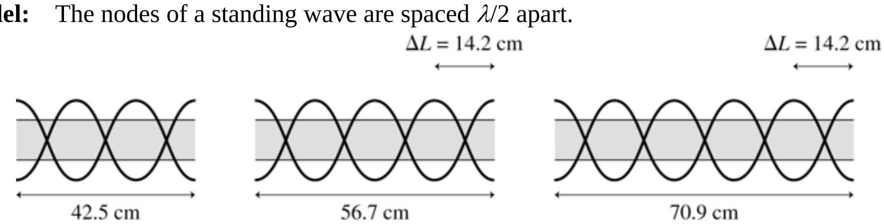
(d) With carbon dioxide, the new fundamental frequency is

$$f_1 = \frac{v}{2L} = \frac{280 \text{ m/s}}{2(1.32 \text{ m})} = 106 \text{ Hz}$$

Thus the frequencies of the $n = 3$, 4, and 5 modes are $f_3 = 3f_1 = 318$ Hz, $f_4 = 4f_1 = 424$ Hz, and $f_5 = 5f_1 = 530$ Hz.

21.50. Model: The nodes of a standing wave are spaced $\lambda/2$ apart.

Visualize:



Solve: The wavelength of the mth mode of an open-open tube is $\lambda_m = 2L/m$. Or, equivalently, the length of the tube that generates the mth mode is $L = m(\lambda/2)$. Here λ is the same for all modes because the frequency of the tuning fork is unchanged. Increasing the length of the tube to go from mode m to mode $m + 1$ requires a length change

$$\Delta L = (m + 1)(\lambda/2) - m\lambda/2 = \lambda/2$$

That is, lengthening the tube by $\lambda/2$ adds an additional antinode and creates the next standing wave. This is consistent with the idea that the nodes of a standing wave are spaced $\lambda/2$ apart. This tube is first increased ΔL = 56.7 cm – 42.5 cm = 14.2 cm, then by ΔL = 70.9 cm – 56.7 cm = 14.2 cm. Thus $\lambda/2$ = 14.2 cm and thus λ = 28.4 cm = 0.284 m. Therefore the frequency of the tuning fork is

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.284 \text{ m}} = 1208 \text{ Hz}$$

21.60. Model: Constructive or destructive interference occurs according to the phases of the two waves.

Solve: The phase difference between the sound waves from the two speakers is

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0$$

With no delay between the two signals, $\Delta\phi_0 = 0$ rad and

$$\Delta\phi = \frac{2\pi(2.0\text{ m})}{v/f} = 2\pi(2.0\text{ m})\left(\frac{340\text{ Hz}}{340\text{ m/s}}\right) = 4\pi\text{ rad}$$

According to Equation 21.22, this corresponds to constructive interference. A delay of 1.47 ms corresponds to an inherent phase difference of

$$\Delta\phi_0 = \left(\frac{2\pi}{T}\right)(1.47\text{ ms})\text{ rad} = (2\pi f)(1.47\text{ ms})\text{ rad} = 2\pi(1.47\text{ ms})(340\text{ Hz})\text{ rad} = \pi\text{ rad}$$

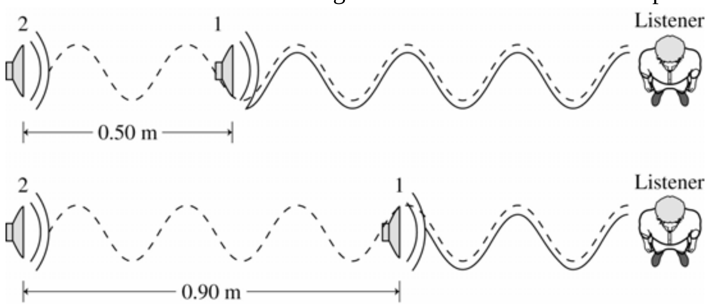
The phase difference $\Delta\phi$ between the signals is then

$$\Delta\phi = 2\pi\left(\frac{\Delta x}{\lambda}\right) + \Delta\phi_0 = 4\pi\text{ rad} + \pi\text{ rad} = 5\pi\text{ rad}$$

Thus, the interference along the *x*-axis will be perfect destructive.

21.61. Model: Interference occurs according to the difference between the phases of the two waves.

Visualize:



Solve: (a) The phase difference between the sound waves from the two speakers is

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0$$

We have a maximum intensity when $\Delta x = 0.50$ m and $\Delta x = 0.90$ m. This means

$$2\pi\frac{(0.50\text{ m})}{\lambda} + \Delta\phi_0 = 2m\pi\text{ rad} \qquad 2\pi\left(\frac{0.90\text{ m}}{\lambda}\right) + \Delta\phi_0 = 2(m+1)\pi\text{ rad}$$

Taking the difference of the above two equations,

$$2\pi\left(\frac{0.40\text{ m}}{\lambda}\right) = 2\pi \Rightarrow \lambda = 0.40\text{ m} \Rightarrow f = \frac{v_{\text{sound}}}{\lambda} = \frac{340\text{ m/s}}{0.40\text{ m}} = 850\text{ Hz}$$

(b) Using again the equations that correspond to constructive interference,

$$2\pi\left(\frac{0.50\text{ m}}{0.40\text{ m}}\right) + \Delta\phi_0 = 2m\pi\text{ rad} \Rightarrow \Delta\phi_0 = \phi_{20} - \phi_{10} = -\frac{\pi}{2}\text{ rad}$$

We have taken $m = 1$ in the last equation. This is because we always specify phase constants in the range $-\pi$ rad to π rad (or 0 rad to 2π rad). $m = 1$ gives $-\frac{1}{2}\pi$ rad (or equivalently, $m = 2$ will give $\frac{3}{2}\pi$ rad).