The Schrodinger-Poisson system is a Schrodinger equation whose potential solves a Poisson equation as follows:

$$
\begin{aligned}
i\Psi_t &= -\frac{1}{2}\Delta\Psi + V\Psi \qquad (1)\\
\Delta V &= \Psi\Psi^* \qquad (2)
\end{aligned}
$$

Using the form of the laplacian in spherical symmetry, the Poisson equation becomes:

$$\frac{2}{r}V_{\mathrm{r}} + V_{\mathrm{rr}} = \Psi\Psi^* \qquad (3)$$

To simplify things, we impose that all functions are zero at a finite outer boundary r = 1,

$$
\begin{aligned}
V|_{r=1} &= 0 \qquad (4)\\
\Psi|_{r=1} &= 0 \qquad (5)
\end{aligned}
$$

and are smooth at the origin,

$$
\begin{aligned}
\frac{dV}{dr}|_{r=0} &= 0 \qquad (6)\\
\frac{d\Psi}{dr}|_{r=0} &= 0 \qquad (7)
\end{aligned}
$$

To discretize the spatial domain into Nx grid points, FORTRAN notation is used, ie:

$$
\begin{aligned}
r_1 &= 0 \qquad (8)\\
r_2 &= h \qquad (9)\\
r_3 &= 2h \qquad (10)\\
&\vdots \qquad (11)\\
r_{\mathrm{Nx}} &= 1 \qquad (12)
\end{aligned}
$$

By introducing the $\mathcal{O}(h^2)$ accurate forward-differencing derivative operator,

$$(f_{\mathrm{x}})^n_j = \frac{-\frac{3}{2}f^n_j + 2f^n_{j+1} - \frac{1}{2}f^n_{j+2}}{h} + \mathcal{O}(h^2) \qquad (13)$$

The inner boundary condition becomes:

$$f^n_1 = -\frac{1}{3}f^n_3 + \frac{4}{3}f^n_2 \qquad (14)$$

where $f$ is either $V$ or $\Psi$. This allows for all references to $V_1$ and $\Psi_1$ to be removed from the the finite-differencing scheme. Using the standard $\mathcal{O}(h^2)$ centered derivative operators, the Poisson equation can be discretized on the inner domain. As an example, with five spatial grid points, the Poisson equation can be written in tridiagonal form:

$$\frac{1}{h^2} * \begin{pmatrix} -2 & \frac{4}{3} & 0 & 0 \\ 1-\frac{h}{r_3} & -2 & 1+\frac{h}{r_3} & 0 \\ 0 & 1-\frac{h}{r_4} & -2 & 1+\frac{h}{r_4} \\ 0 & 0 & 0 & h^2 \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \end{pmatrix} = \begin{pmatrix} \Psi_2\Psi_2^* \\ \Psi_3\Psi_3^* \\ \Psi_4\Psi_4^* \\ \Psi_5\Psi_5^* \end{pmatrix} \qquad (15)$$

The entries in the first row are different because we have used equation 14 to write $V_2$ in terms of $V_2$ and $V_3$.

For the schrodinger equation we apply a forward-difference time derivative operator to the left hand side which is $\mathcal{O}(h^2)$ accurate at $t_{n+\frac{1}{2}}$. To center the rest of the equation at time $t_{n+\frac{1}{2}}$ we appy the forward-time averaging operator to the right hand side. This gives:

$$
\begin{aligned}
i\frac{\Psi^{n+1}_j - \Psi^n_j}{dt} = &- \frac{1}{2r_j}\frac{\left(\Psi^{n+1}_{j+1} - \Psi^{n+1}_{j-1} + \Psi^n_{j+1} - \Psi^n_{j-1}\right)}{2h} \qquad (16)\\
&- \frac{1}{4}\frac{\left(\Psi^{n+1}_{j+1} - 2\Psi^{n+1}_j + \Psi^{n+1}_{j-1} + \Psi^n_{j+1} - 2\Psi^n_j + \Psi^n_{j-1}\right)}{h^2} \qquad (17)\\
&+ \frac{1}{2}\left(V^{n+1}_j\Psi^{n+1}_j + V^n_j\Psi^n_j\right) \qquad (18)
\end{aligned}
$$

Treating the retarded time level as known, and the advanced time level of $V$ as known, this can be brought into tridiagonal form:

$$\frac{(r_j - h)}{4 r_j h^2} \Psi_{j-1}^{n+1} + \left( \frac{i}{dt} - \frac{1}{2h^2} - \frac{1}{2} V_j^{n+1} \right) \Psi_j^{n+1} + \frac{(r_j + h)}{4 r_j h^2} \Psi_{j+1}^{n+1} = + \frac{i}{dt} \Psi_j^n - \frac{1}{4 h r_j} \left( \Psi_{j+1}^n - \Psi_{j-1}^n \right) \tag{19}$$

$$- \frac{1}{4h^2} \left( \Psi_{j+1}^n - 2\Psi_j^n + \Psi_{j-1}^n \right) \tag{20}$$

$$+ \frac{1}{2} V_j^n \Psi_j^n \tag{21}$$

Again applying equation 14 the first equation becomes:

$$\left( \frac{i}{dt} - \frac{1}{2h^2} - \frac{1}{2} V_2^{n+1} \right) \Psi_2^{n+1} + \frac{1}{2h^2} \Psi_3^{n+1} = \frac{i}{dt} \Psi_2^n - \frac{1}{2h^2} \left( \Psi_3^n - \Psi_2^n \right) + \frac{1}{2} V_2^n \Psi_2^n \tag{22}$$