# SPH Hydrodynamics (cont')

The Euler equations ignore viscosity. In some situations (e.g. shocks), viscosity is not negligible.

SPH solution: Add artificial viscosity

Momentum Equation:

$$dv_a/dt = -\sum_b (P_a/\rho_a^2 + P_b/\rho_b^2 + \Pi_{ab}) m_b \nabla W(r_a - r_b, h)$$

Energy Equation:

$$d\epsilon/dt = P_a/\rho_a^2 \sum_b m_b (v_a - v_b) \nabla W(r_a - r_b, h)$$

$$1/2 \sum_b m_b \Pi_{ab} (v_a - v_b) \nabla W(r_a - r_b, h)$$

where $\Pi_{ab} = \begin{cases} (-\alpha c_s \mu_{ab} + \beta \mu_{ab}^2)/\rho & \text{if } (v_a - v_b) \cdot (r_a - r_b) < 0 \\ 0 & \text{otherwise} \end{cases}$

Bulk Viscosity

von Neuman–Richtmeyer viscosity

Conserves Energy!