EVOLUTION OF DYNAMICAL BLACK HOLE SPACETIMES USING EXCISION TECHNIQUES

- Black Hole Excision

- Spherically Symmetric SU(2) Yang Mills Collapse (1+1)
  - "Colored" ("hairy") BH Formation via Fine-Tuning

- Axisymmetric Scalar Collapse (2+1)
  - BH Formation and Growth via Scalar Field Dynamics

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Black Hole Excision

- Fundamental Problem: Black hole spacetimes contain physical singularities–apparently must be avoided at all costs in calculations

- Traditional Approach: Use coordinate freedom (choice of lapse and shift) to "freeze out" evolution in vicinity of physical singularity

- Spherically Symmetric Example: Choose coordinates r, t so that metric takes on "time-dependent-Schwarzschild" form:

$$ds^2 = -\alpha(r,t)^2\, dt^2 + \left(1 - \frac{2m(r,t)}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2$$

  Consider collapse of ball of matter to form black hole: when horizon formation is imminent, then near the Schwarzschild radius $r = R_S$

$$\alpha \to 0 \qquad \left(1 - \frac{2m(r,t)}{r}\right)^{-1} \to \infty$$

  Physical singularity avoided, but coordinate singularity develops, code will crash on the order of black-hole dynamical time, $M = R_S/2$

Black Hole Excision (cont.)

- Excision Approach (**Unruh**): By definition, interior of black hole is out of causal contact with exterior spacetime. Simply remove regions inside event horizons from computational domain.

- Event horizon location not known until after computation is completed, use as inner boundary some surface known to lie within the event horizon (EH).

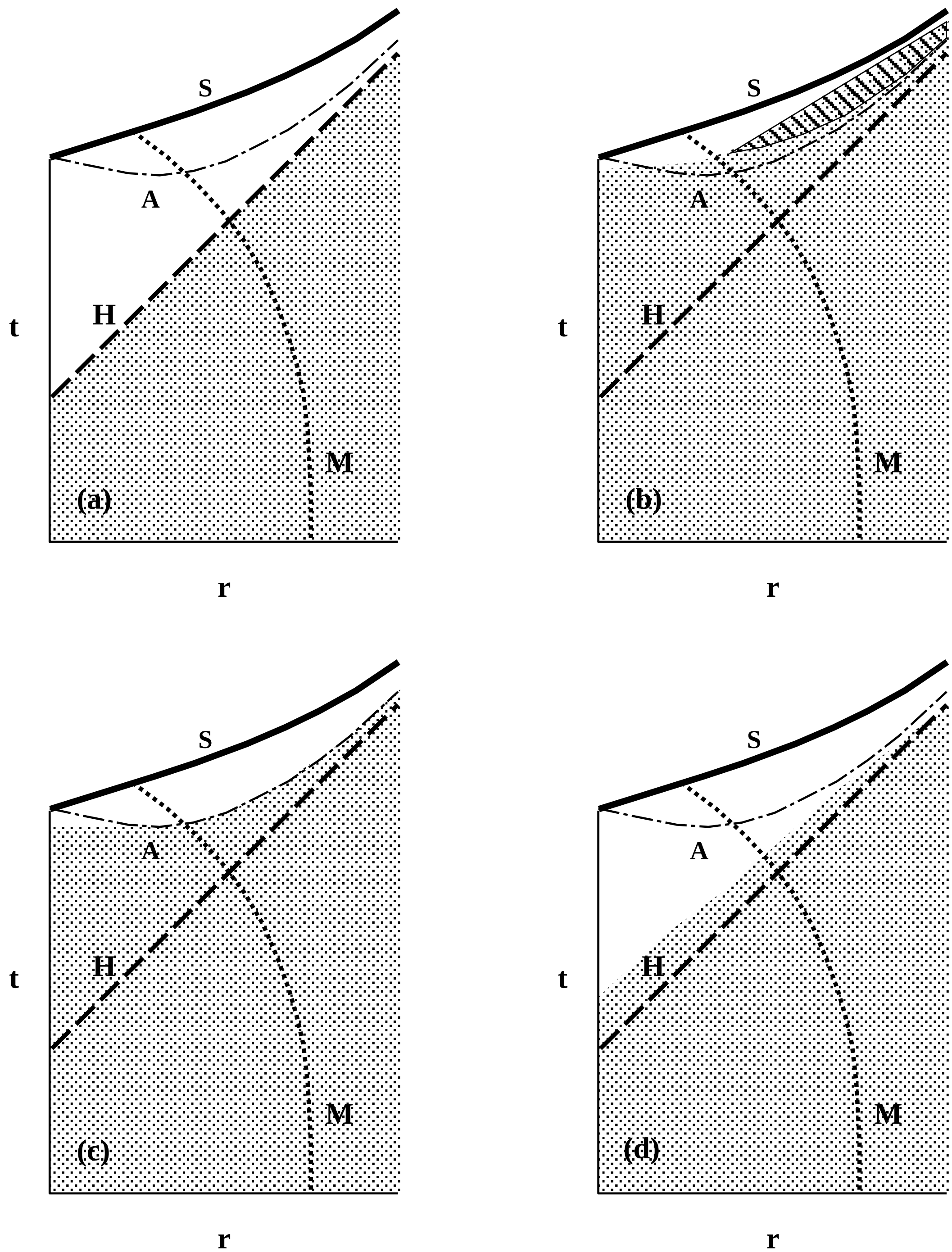
- Apparent horizon (AH): Two surface defined at some instant of time on which divergence of outgoing null rays vanishes— outgoing light rays emanating from surface "hover" at constant radius

- Assuming cosmic censorship, AH will always lie within EH, AH can be located at an instant of time by solving non-linear elliptic equation, but *do not* have to track AH, and/or use AH as cut-off surface.

- Computational boundary is essentially a characteristic (or super-characteristic) surface: modulo possible problems from non-hyperbolicity, don't need boundary conditions

- Efficacy of approach demonstrated in several spherically symmetric calculations starting with **Seidel & Suen (1992)**

Black Hole Excision (cont.)



Spherically Symmetric SU(2) EYM

- SS Scalar Collapse (`Choptuik & Unruh, 1988-1989, unpublished`)
  - Minimally-Modified Ingoing Eddington-Finkelstein
  - Unresolved stability problems

- SS Scalar Collapse
  (`Marsa UT PhD 1995, Marsa & Choptuik, PRD 1996`)
  - Minimally-Modified Ingoing Eddington-Finkelstein
  - Ingoing Eddington-Finkelstein
  - ST must ALWAYS contain black hole, but can grow via scalar accretion
  - Stable with "ad hoc" differencing (read *not* "causal differencing"), explicit dissipation

- SS Scalar Collapse (`Marsa 1997, unpublished`)
  - Use different slicing so that BH FORMATION can be studied as well
  - Maximal Slicing/Areal Spatial Coordinates
  - Stable with "ad hoc" differencing, explicit dissipation

Spherically Symmetric SU(2) EYM (cont.)

(Choptuik, Hirschmann and Marsa, PRD 1999)

- Minimally-coupled, spherically-symmetric SU(2) field (potential), $W(r,t)$ and Einstein field

- Solution via straightforward modification of scalar code; maximal/areal coordinates with black hole excision

- Model known to have interesting phenomenology—including "colored black holes"—*static* BH solutions of arbitrary radius, R_S dressed with non-trivial YM configuration, further parametrized by number, n, of zero crossings of YM potential $W(r)$:

$$W_n(r;\, R_S)$$

- W_n known to have n unstable modes in perturbation theory, W_1 candidate for "critical solution" *a la* "black hole critical phenomena"

- **CHM** verified this conjecture using direct solution of dynamic PDEs, fine-tuning of one-parameter families of initial data

Axially Symmetric Scalar Collapse

(Hirschmann, Liebling, Choptuik, Pretorius, 1998-)

- Minimally coupled (massless) scalar field, coupled to Einstein field, in axisymmetry.
- Use 2+1+1 "Geroch" formalism, previously used by Nakamura et al
- Work in *cylindrical coordinates* (t, ρ, z, φ), rather than *polar spherical* (t, r, θ, φ).
- Conformally flat 2-metric, 3-maximal slicing
- "Fully constrained" evolution: solve *elliptics* for lapse, two components of shift, conformal factor. One (two) second-order equation for "radiative" part of geometry without (with) rotation.
- Stable with "ad hoc" differencing, appropriate definition and regularization of variables, explicit dissipation
- CAVEATS:
    - Code/results testing not yet complete
    - When forming BH from regular initial data, WITHOUT EXCISION, code crashes, "as expected", i.e. when grid point to grid point variation is significant fraction of total dynamical range, i.e. on a dynamical time scale

Axially Symmetric Scalar Collapse (cont.)

Excision Strategy

(Pretorius & Choptuik 2000-)

- Scan level-surfaces (level-curves) of some *a priori* specified, smooth function
$$F(\rho^2, z^2)$$
with spacelike normal s^μ, for which the divergence of the outgoing null field
$$s^\mu + t^\mu$$
is sufficiently negative

- Dirichlet ("frozen") conditions "for constrained" variables at excision surface (inconsistent, but not grossly so, modifications for consistency in the works)

- Low order, one-sided differencing of "evolved" variables EOM at excision surface, plus explicit dissipation (Lax averaging)

Concluding Notes

Excision techniques provide the basis for QUALITATIVE change in our ability to simulate dynamical BH spacetimes

Many details to be worked out for generic 3+1 case; however, these and other results are very encouraging.

Will almost certainly have major impact in theoretical astrophysics in years to come.