Problems of calculating wave signals which characterize isolated systems

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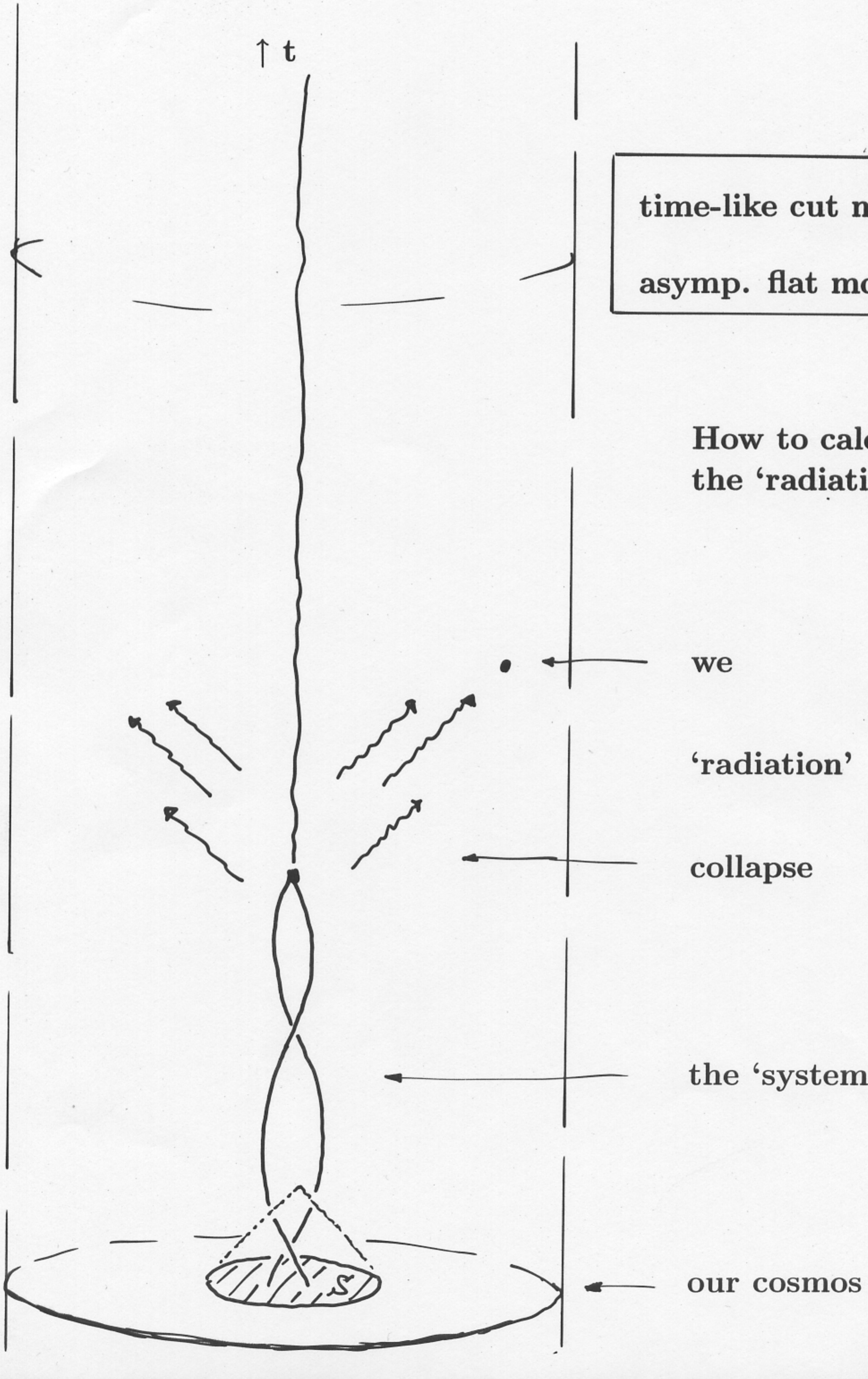
THE GENERAL SITUATION

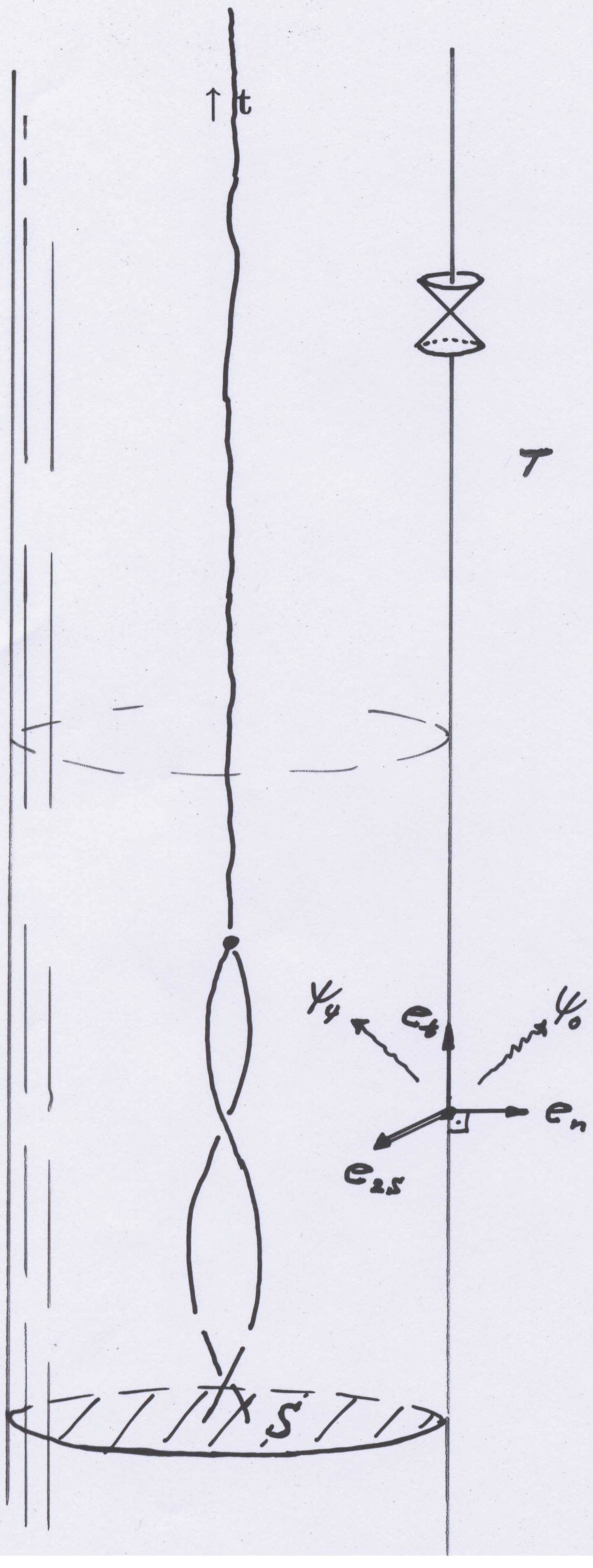
time-like cut model ?

asymp. flat model ?

How to calculate the 'radiation' ?



THE TIME-LIKE CUT MODEL



choose time-like cut T
and space-like section S
study ibvp, $Ric[g] = 0$
near T

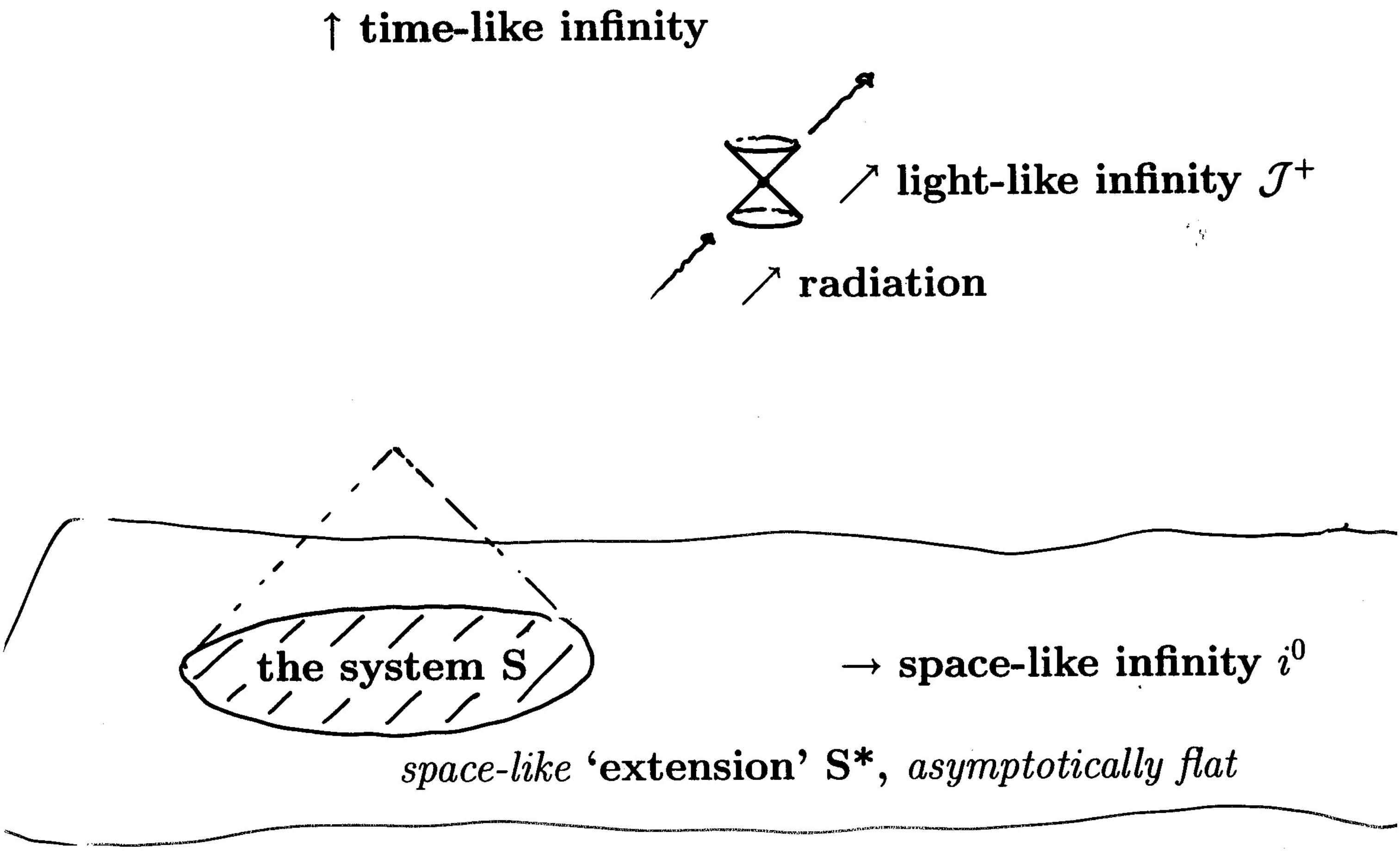
technical problems:

- analytical treatment
  initial data: standard
  bdry data: χ, Ψ_4
- sources
- choice of gauge, life time
- numerical treatment

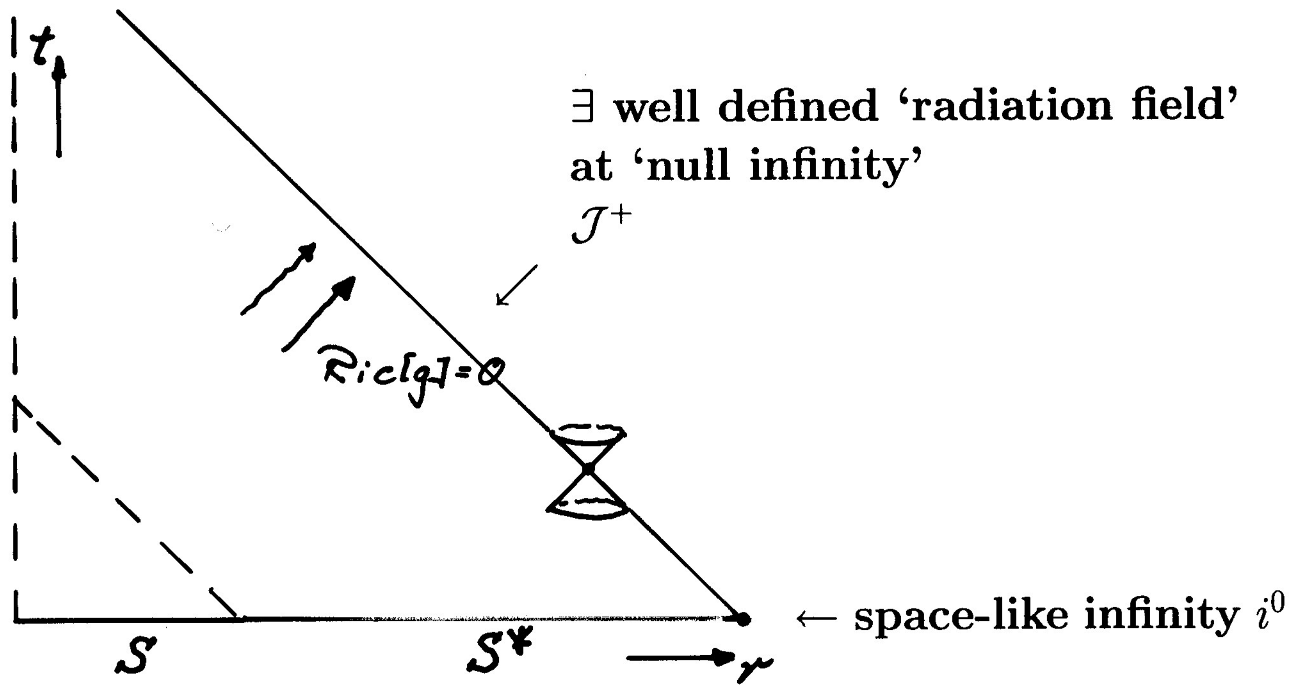
conceptional problems:

- radiation signal = ?
- boundary data = ???

THE ASYMPTOTICALLY FLAT MODEL



2-dim schematic causal picture:

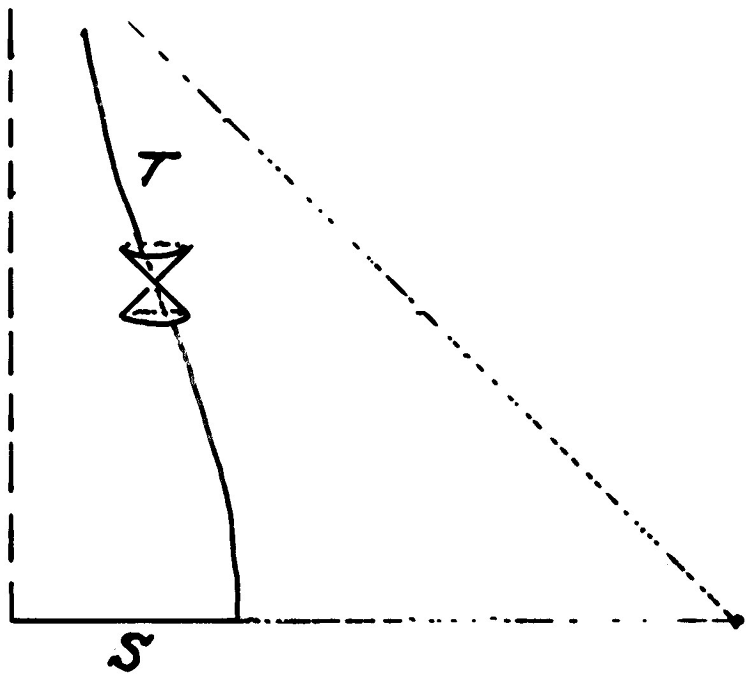


conformal representation: $g \to \tilde{g} = \Omega^2 g, \quad \tilde{M} = M \cup \mathcal{J}^+,$

$\Omega > 0$ **on** M, $\Omega = 0$, $d\Omega \neq 0$ **on** $\mathcal{J}^+$

APPROACHES I

Standard approach:



introduce T
to get finite
comp. domain

data on S ∪ T

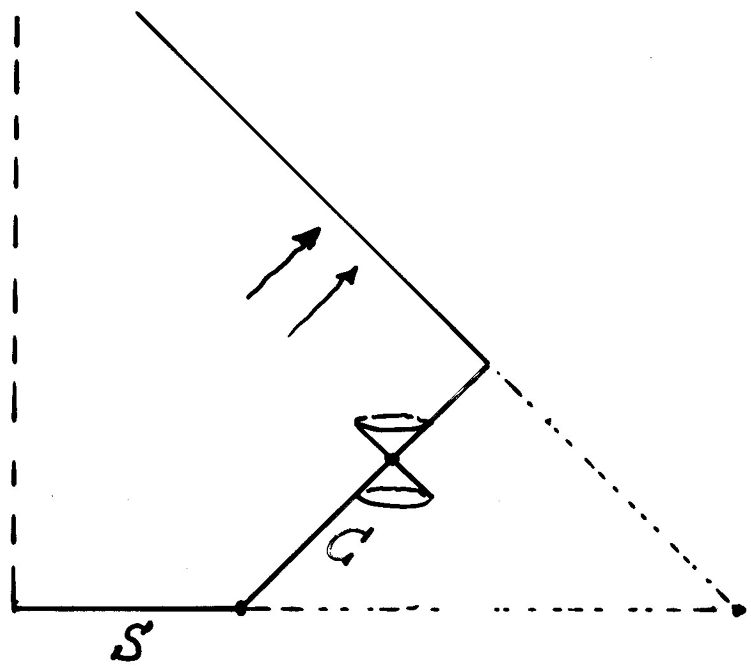
based on 3+1

– radiation signal = ?

– boundary data = ?

– 'as. flat' helps ?

Characteristic approach:



introduce char. C
perform coordinate
compactification

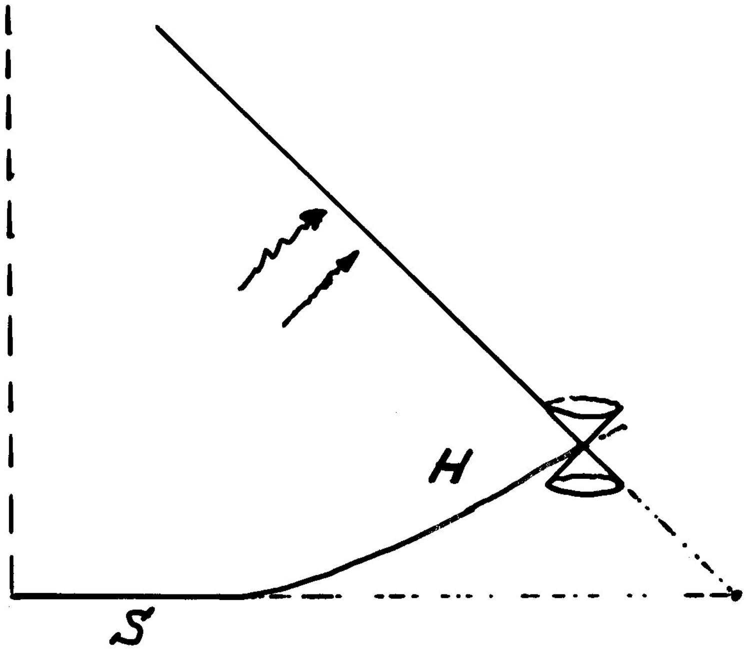
data on S ∪ C

based on null foliation
(and 3 + 1)

calculate radiation
field on $\mathcal{J}^+$!

APPROACHES II

Hyperboloidal approach:



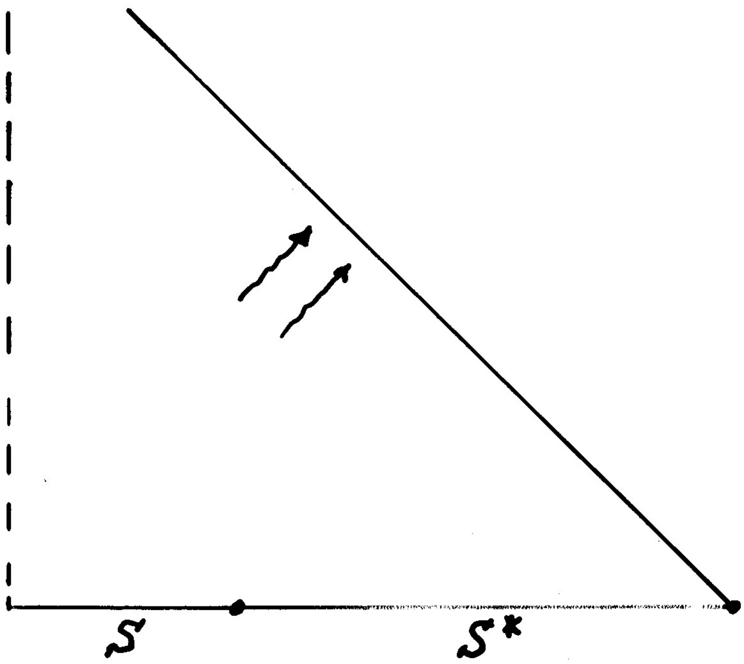
introduce H
use conformal
picture and
equations

data on S ∪ H

based on 3+1 or ...

calculate radiation
field on $\mathcal{J}^+$!

Global approach:



use conformal
picture and
equations

data on S ∪ S*

based on 3+1 or ...

calculate the entire
solution and the
radiation field on $\mathcal{J}^+$!

not done yet !

PROBLEM COMMON TO ALL APPROACHES

How does one choose the data on T, C, H and on S' so as to calculate signals characterizing the 'system' and not the data ?

How do the calculated signals depend on the choice of data on T, C, H and on S' ?

$\exists$ physical arguments for preferred choices ?

Not clear whether *'the spurious radiation will go out quickly and the basic signal will remain'*

Do there exist characteristics of the wave signals which are intrinsic to the 'system' and independent of the choice of extensions ?

$\exists$ analytical arguments for preferred choices ?

Not clear yet, but ...

POSSIBLE CRITERIA ?

S. Dain (2004):

With a given asymptotically flat initial data set for Einstein's field equations can be associated (*besides the mass*) a geometric invariant which vanishes if and only if the data are stationary.

- Obtained by solving a PDE of order 4.
- In vacuum this invariant can be interpreted as a measure of the radiation content.
- Possible to minimize this invariant in suitable classes of data ?

H.F. (1998, 2004), J. Valiente-Kroon (2004):

For time-symmetric asymptotically flat initial data the requirement that $\mathcal{J}^+$ be smooth (up to a certain order) appears to imply that the data are asymptotically static (up to a certain order) and vice versa.

- Evidence appears to be irrefutable.
- $\exists$ large classes of such data, general in the interior Corvino-Schoen (2000, 2003), Delay-Chruściel (2002, 2003).
- Possible generalizations under investigation.
- Asymptotic stationarity too strong/enough/too weak ?